THE

Complete Meaſurer.

Ld OR, THE Doneraile

Whole Art of Meaſuring.

In Two Parts.

PART I.

Teaching Decimal Arithmetic, with the Extraction of the Square and Cube-Roots.

Alſo the Multiplication of Feet and Inches, commonly called *Croſs-Multiplication.*

PART II.

Teaching to meaſure all Sorts of Superficies and Solids, by *Decimals*, by *Croſs Multiplication*, and by *Scale* and *Compaſſes:* Alſo the Works of ſeveral Artificers relating to Building; and the meaſuring of Board and Timber; ſhewing the common Errors. And ſome Practical Queſtions.

The Fifth Edition.
To which is added an Appendix, 1. Of *Gaging.*
2. Of *Land Meaſuring.*

Very uſeful for all Tradeſmen, eſpecially Carpenters, Bricklayers, Plaiſterers, Painters, Joyners, Glaſiers, Maſons, *&c.*

By William Hawney, *Philomath.*

Recommended by the Rev. Dr John Harris. F.R.S.

DUBLIN:
Printed by and for I. Jackson in *Meath-ſtreet.*

THE
Compleat Measurer:
OR, THE
Whole Art of Measuring.

In Two Parts.

PART I.

Teaching Decimal Arithmetick, with the Extraction of the Square and Cube Roots.

Also a [illegible] [illegible] of [illegible] [illegible], commonly called [illegible] [illegible].

PART II.

Teaching to measure all Sorts of Superficies and Solids, by Decimals, by Gunter's Line, and by Vulgar and Cross Multiplication. Also the Works of the several Artificers relating to Building; and the measuring of Board and Timber; shewing the common Errors [illegible] [illegible] Questions.

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To which is added an Appendix, 1. Of Gauging. 2. Of Land Surveying.

Adapted for the Use of Workmen, especially Carpenters, Bricklayers, Plaisterers, Painters, Joyners, Glaziers, Masons, &c.

By WILLIAM HAWNEY, [illegible]

Recommended [illegible] [illegible] [illegible]

DUBLIN:
Printed by and for I. Jackson in [illegible]

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THE PREFACE.

HAVING peruſed ſeveral Books concerning the Menſuration of Superficies and Solids, and the Works of Artificers relating to Building; but not finding any one Book ſo perfect, as to give any tolerable Satiſfaction to a Learner; and I having practiſed and taught Meaſuring for ſeveral Years, and thereby gained Experience and Knowledge in that Art, having learned ſome Things from one Author, and ſome Things from another, I began to think of digeſting my

Thoughts into ſome ſuch Method as might give a Learner full Satisfaction, without being at the Charge of buying ſo many Books; and being importuned thereunto by ſome Friends, I fell to work, and at laſt brought them to that Perfection you here find the following WORK.

1. As to the DECIMAL ARITHMETIC, I have been as brief as the Matter would well bear, to make it plain.

2. As to the multiplying of Feet and Inches, commonly called CROSS-MULTIPLICATION, my Method differs from that which is uſually taught in other Authors, as being (I think) much ſhorter and plainer.

3. In meaſuring of Superficies and Solids, I have given the Demonſtration of the Rules, which I thought might be very acceptable to the Ingenious; for indeed I always look upon the writing of a Rule without a Demonſtration, (in any Part of the Mathematicks) to be but lame and defective; and for want of knowing the Reaſon of the Rule, a

Learner may commit great Errors: Beſides, when a Learner knows the Reaſon of the Rules, he may retain them better in his Memory. The Rule for meaſuring a Priſmoid and Cylindroid, I had out of Mr. *Everard*'s *Art of Gaging*; but the Reaſon he does not ſhew, neither have I found it in any other Author; but that the Method is true, I have endeavoured to make plain.

The Demonſtrations of the Rules for finding the Area of an Ellipſis and Parabola; alſo the Demonſtration of the Rules for finding the ſolid Content of the Fruſtum of a Cone and Pyramid, the Solidity of a Globe, of a Spheroid, a Parabolick Conoid, and of a Parabolick Spindle, and their Fruſtums, I had from the ingenious Mr. *Ward*'s *Young Mathematician's Guide*; where the curious and ingenious Reader may ſee many other Demonſtrations algebraically perform'd: I have alſo demonſtrated the Rule for finding the Solidity of a Globe, out of *Pardie*'s *Elements of Geometry*, (Book the Vth. Art. the 33d.) publiſhed in *Engliſh*, with many Additions, by the Reverend Dr. *Harris*, F.R.S. and the ſame is alſo done out of *Sturmius*'s *Matheſis Enucleata*; ſo that the ingenious Reader may uſe which of thoſe Ways he likes beſt.

The Scale ſuppoſed to be uſed in all the Operations is the Line of Numbers, commonly called *Gunter's Line*, which is upon the ordinary two Feet or eighteen Inch Rules, commonly uſed by the Carpenters, Maſons, *&c.* becauſe I thought it needleſs, as well as impertinent, to write the Uſe of Sliding-Rules, or any other particular Scales, they being ſufficiently treated of by ſeveral Authors, *viz.* by the above-named Mr. *Everard*, in his *Art of Gaging*, abovementioned, where you have the Uſe of a Sliding-Rule in Arithmetic, Geometry, in meaſuring of Superficies and Solids, Gaging, *&c.* Likewiſe Mr. *Hunt* has wrote largely of the Uſe of his Sliding-Rule, in Arithmetic, Geometry, Trigonometry, Gaging, Dyalling, *&c.* There are ſeveral others who have explained the Uſe of their own Rules; ſo that the more curious Readers may find full Satisfaction in thoſe Authors.

One Thing I have omitted in the Book, which I think may not be very improperly inſerted in this Place; that is, how to find a Number upon the Line. If the Number you would find conſiſts only of Units, then the Figures upon the Line repreſent the Number ſought: Thus, if the Number be 1, 2, 3, *&c.* then 1, 2, 3, *&c.* upon the Line, repreſent the Number ſought; but if the Number conſiſts of two Figures, that is, of Units and Tens, then the Figure upon the Rule ſtands for Tens, and the larger Diviſions ſtand for Units: Thus, if 34 were to be found upon the Line, the Figure 3 upon the Line is 30, and 4 of the large Diviſions (counted forward) is the Point repreſenting 34; and if 340 were to be found, it will be at the ſame Point upon the Line; and if 304 were to be found, then the 3 upon the Line is 300, and 4 of the ſmaller Diviſions (counted forward) is the Point repreſenting 304: If the Number conſiſts of four Places, or Thouſands, then the Figure upon the Line ſtands for Thouſands, and the larger Diviſions are Hundreds, the leſſer Diviſions are Tens, and the tenth Parts of thoſe leſſer Diviſions are Units; thus, if 2735 were to be found, then the 2 is 2000, and the 7 larger Diviſions (counted forward) is 700 more, and 3 of the leſſer Diviſions is 30 more, and half of one of the leſſer Diviſions is 5 more, which is the Point repreſenting 2735. You muſt remember, that between each Figure upon the Line there are ten Parts, which I call the larger Diviſions, and each of thoſe larger Diviſions are ſubdivided (or ſuppoſed ſo to be) into ten other Parts, which I call the ſmaller Diviſions, and each of thoſe Parts ſuppoſed to be ſubdivided again into ten other Parts, *&c.* You muſt alſo remember, that if 1, in the Middle of the Line, ſtands only for 1, then 1 at the upper End will be 10, and 1 at the lower End will only be $\frac{1}{10}$; but if 1 at the lower End ſignifies 1, then 1 in the Middle ſtands for 10, and 1 at the upper End is 100, *&c.*

There is one Thing more which I would have my Reader to underſtand, and that is, How to find all ſuch proportional Numbers made Uſe of in the Proportions about a Circle, and of a

Cylinder, and in other Places; which Thing may be of good Uſe, to know how to correct a Number which may happen to be falſe printed, or to enlarge any Number to more decimal Places, for more Exactneſs; for though I have mentioned what ſuch Numbers are, yet I have not ſhewn how to find them, which a Learner may be a little at a Nonplus to do, though they are eaſily found by the Rules there laid down. I ſhall therefore give two or three Examples, in this Place, of finding ſuch Numbers, which may enable my Reader to find out the reſt.

And, firſt, let it be required to find the Area of a Circle, whoſe Diameter is an Unit.

By the Proportion of *Van Culen*, if the Diameter be 1, the Circumference will be 3.1415926, *&c.* whereof 3.1416 is ſufficient in moſt Caſes. Then the Rule teaches to multiply half the Circumference by half the Diameter, and the Product is the Area; that is, multiply 1.5708 by .5, (*viz.* Half 3.1416 by Half 1) and the Product is .7854, which is the Area of the Circle, whoſe Diameter is 1.

Again, if the Area be required, when the Circumference is 1; firſt, find what the Diameter will be, thus, As 3.1416 : to 1, :: ſo is 1 : to .318309, which is the Diameter when the Circumference is 1. Then multiply half .318309 by half 1, that is, .159154 by .5, and the Product is .079577, which is the Area of a Circle whoſe Circumference is 1.

If the Area be given to find the Side of the Square equal, you need but extract the Square Root of the Area given, and it is done: So the Square Root of .7854 is 8862, which is the Side of a Square equal when the Diameter is 1. And if you extract the Square Root of .079577, it will be .2821, which is the Side of the Square equal to the Circle, whoſe Circumference is 1.

If the Side of a Square within a Circle be required, if you ſquare the Semidiameter and double that Square, and out of that Sum extract the Square Root, that ſhall be the Side of the Square which may be inſcribed in that Circle; ſo, if the Diameter of the Circle be 1, then the Half is .5; which ſquared is .25, and this doubled is .5, whoſe Square Root is .7071, the Side of the Square inſcribed.

Again, if the Diameter of a Globe be 1, to find the Solidity. In Sect. XI. Chap. II. it is demonſtrated, that the Globe is ⅔ of a Cylinder of the ſame Diameter and Altitude: Thus, if the Cylinder's Diameter be 1, and its Altitude or Length be alſo 1, find the Solidity thereof, and take ⅔ of it, and that will be the Solidity of the Globe required. Now, if the Diameter be 1, the Area of the Circle or Baſe of the Cylinder, is .7854 (as is above ſhewn) which multiplied by 1, the Altitude of the Cylinder, and the Product is alſo .7854, the Solidity of the Cylinder, ⅔ whereof is .5236, which is the Solidity of the Globe whoſe Diameter is 1.

From what has been ſaid, the Reader may eaſily perceive how all other proportional Numbers are ſound, and may examine them at his Pleaſure.

I ſhall not enlarge any farther upon the Matter, but leave the Book to ſpeak for itſelf; and if it prove beneficial to the ingenious Practitioners, I have my Deſire. So, wiſhing my ingenious Reader good Succeſs in his Endeavours, not doubting but he will reap Profit hereby; which that he may, is the hearty Deſire of his Well-wiſher,

W. HAWNEY.

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Complete Meaſurer.

PART I.

CHAP. I.

Notation of DECIMALS.

A Decimal Fraction is an artificial Way of ſetting down and expreſſing of Natural or Vulgar Fractions, as whole Numbers: And whereas the Denominators of Vulgar Fractions are diverſe, the Denominators of Decimal Fractions are always certain: For a Decimal Fraction hath always for its Denominator an Unit, with a Cypher or Cyphers annexed to it, and muſt therefore be either 10, 100, 1000, 10000, *&c.* And therefore, in writing down of a Decimal Fraction there is no Neceſſity of writing down the Denominator; for by bare Inſpection it is certainly known; it conſiſting of an Unit, with as many Cyphers annexed to it as there are Places (or Figures) in the Numerator.

Example. This Decimal Fraction $\frac{25}{100}$ may be written thus, .25, its Denominator being known to be an Unit with two Cyphers; becauſe there are two Figures in the Numerator. In like manner, $\frac{125}{1000}$ may be thus written, .125, $\frac{3575}{10000}$ thus, .3575; and $\frac{75}{1000}$ thus, .075, and $\frac{65}{10000}$ thus, .0065.

As whole Numbers increaſe in a Decuple, or tenfold Proportion, towards the Left Hand, ſo, on the contrary, Decimals decreaſe, towards the Right Hand, in a decuple Proportion, as in the following Scheme.

Tens of Millions.	Millions.	Hundreds of Thouſands.	Tens of Thouſands.	Thouſands.	Hundreds.	Tens.	Units.	Tenth Parts.	Hundredth Parts.	Thouſandth Parts.	Ten Thouſandth Parts.	Hundred Thouſandth Parts.	Millionth Parts, *&c.*
7	6	5	4	3	2	1	0	1	2	3	4	5	6

Hence it appears, that Cyphers put on the Right Hand of whole Numbers, do increaſe the Value of thoſe Numbers in a decuple (or tenfold) Proportion; but being annexed to the Right Hand of a Decimal Fraction, do neither increaſe nor decreaſe the Value thereof: So $\frac{2500}{10000}$ is equivalent to $\frac{25}{100}$ or .25. And, on the contrary, tho' in whole Numbers, Cyphers prefixed before them do neither increaſe nor diminiſh the Value; yet Cyphers before a Decimal Fraction, do diminiſh its Value in a decuple Proportion: For .25, if you prefix a Cypher before it, becomes $\frac{025}{1000}$, or .025; and .125 is $\frac{00125}{100000}$, by prefixing two Cyphers before it, thus, .0025. And therefore, when you are to write a Decimal Fraction, whoſe Denominator hath more Cyphers than there are Figures in the Numerator, they muſt be ſupplied by prefixing ſo many Cyphers before the Figures of your Numerator; as ſuppoſe $\frac{19}{1000}$ were to be written down without its Denominator; here, becauſe there are three Cyphers in the Denominator, and but two Figures in the Numerator, therefore prefix a Cypher before 19, and ſet it down thus, .019.

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The Integers are ſeparated from the Decimals ſeveral Ways, according to Mens Fancies; but the beſt and moſt uſual Way is by a Point or Period; and if there be no whole Number, then a Point before the Fraction is ſufficient: Thus, if you were to write down $317\frac{217}{1000}$, it may be thus expreſſed, 317.217; and $59\frac{25}{10000}$ thus, 59.0025; and $\frac{75}{10000}$ thus, .0075, *&c.*

CHAP. II.

Reduction of DECIMALS.

IN *Reduction of Decimals*, there are three Caſes; *1ſt*, To reduce a Vulgar Fraction to a Decimal. *2dly*, To find the Value of a Decimal in the known Parts of Coin, Weights, Meaſures, *&c.* *3dly*, To reduce Coin, Weights, Meaſures, *&c*, to a Decimal. Of theſe in their Order.

I. *To reduce a Vulgar Fraction to a Decimal.*

The RULE.

As the Denominator of the given Fraction is to its Numerator, ſo is an Unit (with a competent Number of Cyphers annexed) to the Decimal required.

Therefore, if to the Numerator given, you annex a competent Number of Cyphers, and divide the Reſult by the Denominator, the Quotient is the Decimal equivalent to the Vulgar Fraction given.

Example 1. Let $\frac{3}{4}$ be given, to be reduced to a Decimal of two Places, or having 100 for its Denominator

To 3 (the Numerator given) annex two Cyphers, and it makes 300; which divide by the Denominator 4; and the Quotient is .75, the Decimal required, and is equivalent to $\frac{3}{4}$ given.

NOTE, That ſo many Cyphers as you annex to the given Numerator, ſo many Places muſt be pricked off in the Decimal found; and if it ſhall happen that there are not ſo many Places of Figures in the Quotient, the Deficiency muſt be ſupplied, by prefixing ſo many Cyphers before the Quotient-Figures, as in the next *Example*.

Example 2. Let $\frac{3}{573}$ be reduced to a Decimal having ſix Places.

To the Numerator annex ſix Cyphers, and divide by the Denominator, and the Quotient is 5235; but it was required to have ſix Places, therefore you muſt prefix two Cyphers before it, and then it will be .005235, which is the Decimal required, and is equivalent to $\frac{3}{573}$.

See the WORK of theſe two *Examples*.

```
4)3.00(.75            573)3.000000(5235
  28                         ....
 -----                  -----------
  20                     1350
  20                      2040
 -----                     3210
  ..                    -----------
                            345
```

In the ſecond *Example* there remains 345, which Remainder is very inſignificant, it being leſs than $\frac{1}{1000000}$ Part of an Unit, and therefore is rejected.

II. *To find the Value of a Decimal in the known Parts of Money, Weight, Meaſures, &c.*

The RULE.

Multiply the given Decimal by the Number of Parts in the next inferior Denomination, and from the Product prick off ſo many Places to the Right Hand as there were Places in the Decimal given; and multiply thoſe Figures pricked off by the Number of Parts in the next inferior Denomination, and prick off ſo many Places as before, and ſo continue to do, till you have brought it to the loweſt Denomination required.

Example I. Let .7565 of a Pound Sterling be given to be reduced to Shillings, Pence and Farthings,

Multiply by 20, by 12, and 4, as the Rule directs, and always prick off four Places to the Right Hand, and you will find it to make 15 *s*. 1 *d*. 2 *q*.

See the Work,

.7565
20
s. ------
15.1300
12
d. ------
1.5600
4
q. ------
2.2400

A more compendious Way of finding the Value of the Decimal of a Pound Sterling.

Double the firſt Figure, (or Place of Primes) and it makes ſo many Shillings; and if the next Figure (or Place of Seconds) be 5, or more than 5, for the 5 add another Shilling to the former Shillings; then for every Unit in the ſecond

Place count ten, and to that add the Figure in the third Place, and reckon them ſo many Farthings; but if they make above 13, abate 1, and if it be above 38, abate 2, and add the remaining Farthings to the Shillings before found.

Example 1. Let .695 of a Pound be reduced to Shillings; Pence, and Farthings.

Firſt, Double your 6, and it makes 12 *s.* then take 5 out of 9, and for that reckon another Shilling, and it makes 13 *s.* and the 4 remaining is 4 Tens, and the 5 makes 45, which being above 38, you muſt therefore caſt away 2, and there reſt 43 Farthings, which is 10 *d.* ¾. So the Anſwer is 13 *s.* 10 *d.* ¾.

	l.	*s.*	*d.*
So the Value of	.725=	14	6
And the Value of	.878=	17	6¾
And the Value of	.417=	8	4

And ſo of any other.

Let .59755 of a Pound Troy be reduced to Ounces, Penny-weights, and Grains.

Multiply by 12, by 20, and by 24, and always prick off five Places towards the Right Hand, and you will find the Anſwer to be 7 *oz.* 3 *pwt.* 10 *gr. ferè.*

See the Work.

```
 .59755
     12
-------
7.17060
     20
-------
3.41200
     24
-------
 164800
 82400
-------
9.88800
```

oz. pwt. gr.
Facit 7 3 9.888.

Let .43569 of a Tun be reduced to Hundreds, Quarters, and Pounds.

Multiply by 20, by 4, and by 28, and the Anſwer will be 8 *C.* 2 *qrs.* 14 *lb. ferè.*

```
 .43569
     20
 -------
 8.71580         C. qrs. lb.
      4    Facit 8  2   23.9456.
 -------
 2.85520
     28
 -------
23.94560
```

Let .9595 of a Foot be reduced into Inches and Quarters.

```
  .9595
     12
 -------
11.5140  Facit 11 Inches, 2 Quarters.
      4
 -------
 2.0560
```

III. *To reduce the known Parts of Money, Weight, Meaſure,* &c. *to a Decimal.*

The RULE.

To the Number of Parts of the leſſer Denomination given, annex a competent Number of Cyphers, and divide by the Number of ſuch Parts that are contained in the greater Denomination, to which the Decimal is to be brought; and the Quotient is the Decimal ſought.

Example 1. Let 6 *d.* be reduced to the Decimal of a Pound.

To 6 annex a competent Number of Cyphers (ſuppoſe 3) and divide the Reſult by 240 (the Pence in a Pound) and the Quotient is the Decimal required.

240)6.000(.025
1200
Facit .025
....

Example 2. Let 3 *d.* $\frac{3}{4}$ be reduced to the Decimal of a Pound, having ſix Places.

In 3 *d.* $\frac{3}{4}$ there are 15 Farthings; therefore to 15 annex ſix Cyphers (becauſe there are to be ſix Places in the Decimal required) and divide by 960 (the Farthings in a Pound) and the Quotient is .015625.

96|0)15.000000(.015625.
.....
540
600
240
480
...

Example 3. Let 3 $\frac{1}{4}$ Inches be reduced to the Decimal of a Foot, conſiſting of four Places.

In 3 $\frac{1}{4}$ Inches there are 13 Quarters; therefore to 13 annex four Cyphers, and divide by 48, (the Quarters in a Foot) and the Quotient is .2708.

48)13.0000(.2708
340
400
16

Example 4. Let 9 *C.* 1 *qr.* 16 *lb.* be reduced to the Decimal of a Tun, having ſix Places.

C. qr. lb.	
9 1 16	224|0)1052.000000'0(.469642
4	.....
37 *qrs.*	15600
28	21600
	14400
302 *Facit* .469642	9600
75	6400
1052 Pounds.	1920

CHAP. III.

Addition of DECIMALS.

ADDITION of Decimals is performed the ſame Way as Addition of whole Numbers, only you muſt obſerve to place your Numbers right, that is, Units under Units, Primes under Primes, Seconds under Seconds, *&c.*

Example. Let 317.25, 17.125, 275.5, 47.3579, and 12,75 be added together into one Sum.

317.25
17.125
275.5
47.3579
12.75

Sum 669.9829

This is ſo plain, that more Examples I think needleſs.

CHAP. IV.

Subtraction of DECIMALS.

SUBTRACTION of Decimals is performed likewiſe the ſame Way as in whole Numbers, Reſpect being had to the right placing the Numbers, (as in Addition) as in the following *Examples*.

	(1)		(2)
From	212.0137	From	201.1250
Subtr.	31.1275	Subtr.	5.5785
Reſts	180.8862	Reſts	195.5465
Proof	212.0137	Proof	201.1250

	(3)		(4)
From	2051.315	From	30.5
Subtr.	79.172	Subtr.	7.2597
Reſts	1972.143	Reſts	23.2403
Proof	2051.315	Proof	30.5

NOTE, If the Number of Places in the Decimals be more in that which is to be ſubtracted, than in that which you ſubtract from, you muſt ſuppoſe Cyphers to make up the Number of Places; as in the fourth *Example*.

CHAP.

CHAP. V.

Multiplication of DECIMALS.

MULTIPLICATION of Decimals is alſo performed the ſame Way as Multiplication of whole Numbers; but to know the Value of the Product, obſerve this

RULE.

" Cut off, or ſeparate by a Comma, or Prick, ſo many
" Decimal Places in the Product, as there are Places of Deci-
" mals in both Factors, *viz.* in the Multiplicand and Multipli-
" er, which I ſhall farther explain in the following *Examples*.

Let .3125 be multiplied by 2.75; multiply the Numbers together, as if they were whole Numbers, and the Product is 8.59375: And becauſe there were three Places of Decimals pricked off in the Multiplicand, and two Places in the Multiplier, therefore you muſt prick off five Places of Decimals in the Product, as you may ſee by the Work.

```
   3.125
    2.75
 -------
   15625
  21875
 6450
 -------
 8.59375
```

Let 79.25 be multiplied by .459.

In this *Example*, becauſe two Places of Decimals are prick'd off in the Multiplicand, and three in the Multiplier, therefore there muſt be five prick'd off in the Product.

```
  79.25
   .459
-------
  71325
 39625
31700
-------
36.37575
```

Let .135272 be multiplied by .00425.

In this *Example*, becauſe in the Multiplicand are ſix Decimal Places, and in the Multiplier five Places, therefore in the Product there muſt be eleven Places of Decimals; but when the Multiplication is finiſhed, the Product is but 57490600, *viz.* only eight Places; therefore, in this Caſe, you muſt prefix three Cyphers before the Product Figures, to make up the Number of eleven Places; ſo the true Product will be .00057490600.

```
     .135272
      .00425
------------
      676360
     270544
    541088
------------
.00057490600
```

More Examples for Practice.

.001472
.1045
7360
5888
1472
.0001538240

.017532
347
122724
70128
52596
6.083604

279.25
.445
139625
111700
111700
124.26625

32.0752
0.325
163760
641504
962256
1.04244400

4.443
15.98
35544
39987
22215
4443
70.99914

20.0291
35.45
1001455
801164
1001455
600873
710.031595

7.3564
.0126
441384
147128
73564
.09269064

.75432
.0356
452592
377160
226296
.026853792

C

Con-

Contracted Multiplication of Decimals.

Becauſe in Multiplication of Decimal Parts and mixed Numbers, there is no need to expreſs all the Figures of the Product, but, in moſt Caſes, two, three, or four Places of Decimals will be ſufficient; therefore, to contract the Work, obſerve this following

RULE.

Write the Unit's Place of the Multiplier under that Place of the Multiplicand, whoſe Place you intend to keep in the Product; then invert the Order of all the other Figures, that is, write them all the contrary Way: Then, in multiplying, always begin at that Figure in the Multiplicand which ſtands over the Figure you are then multiplying withal, and ſet down the firſt Figure of each particular Product directly one under the other; but yet a due Regard muſt be had to the Increaſe ariſing from the Figures on the Right Hand of that Figure in the Multiplicand which you begin to multiply at. This will appear more plain by *Examples.*

Example 1. Let 2.38645 be multiplied by 8.2175, and let there be only four Places retained in the Decimals of the Product.

Firſt, according to the Directions, write down the Multiplicand, and under it write the Multiplier, thus; place the 8 (being the Units Place of the Multiplier) under 4, the fourth Place of Decimals in the Multiplicand, and write the reſt of the Figures quite contrary to the uſual Way, as in the following Work: Then begin to multiply, firſt the 5 which is left out, (only with Regard to the Increaſe which muſt be carried from it) ſaying, 8 times 5 is 40, carry 4 in your Mind, and ſay, 8 times 4 is 32, and 4 I carry, is 36; ſet down 6 and carry 3; and proceed thro' the reſt of the Figures, as in common Multiplication: Then begin to multiply with 2, ſaying, 2 times 4 is 8, for which I carry 1, (becauſe it is above 5) and ſay, 2 times 6 is 12, and 1 that I carry, is 13; ſet down 3 and carry 1, and proceed thro' the reſt of the Figures: Then multiply with 1, ſaying, once 6 is 6, for which carry 1, and ſay, once 8 is 8, and 1 is 9; ſet down 9, and proceed:

Then multiply with 7, ſaying, 7 times 8 is 56, for which carry 6, (becauſe it is above 55) and ſay, 7 times 3 is 21, and 6 that I carry, is 27; ſet down 7 and carry 2, and proceed: Then multiply with 5, ſaying, 5 times 3 is 15, for which carry 2, and ſay, 5 times 2 is 10, and 2 I carry, is 12, which ſet down, and add all the Products together, and the total Product will be 19.6107.

See the **Work**.

2.38645
5712.8
190916
4773
239
167
12
19.6107

NOTE, That in multiplying the Figure left out every time next the Right Hand in the Multiplicand, if the Product be 5, or upwards to 10, you carry 1; and if it be 15, or upwards to 20, carry 2; and if 25, or upwards to 30, carry 3, *&c.*

I have here ſet down the Work of the laſt *Example*, wrought by the common Way, by which you may ſee both the Reaſon and Excellency of this Way, all the Figures on the Right Hand of the Line being wholly omitted.

2.386	45
8.21	75
11	93225
167	0515
238	645
4772	90
190916	0
19.6106	51875

Example 2. Let 375.13758 be multiplied by 16.7324, ſo that the Product may have but four Places of Decimals.

Firſt ſet 6, the Units Place of the Multiplier, under 5, being the fourth Place of Decimals in the Multiplicand, (becauſe four Places of Decimals were to be pricked off) and write all the reſt of the Figures backward; then multiply all the Figures of the Multiplicand by 1, after the common Way; then begin with the ſecond Figure of the Multiplier 6, ſaying, 6 times 8 is 48, for which I carry 5; (in reſpect of the 8 left out) and 6 times 5 is 30, and 5 that I carry is 35; ſet down 5 and carry 3, and proceed after the common Method: Then begin with 7, the third Figure of the Multiplier, and ſay, 7 times 5 is 35, for which carry 4, and ſay, 7 times 7 is 49, and 4 I carry is 53; ſet down 3 under the firſt and carry 5, and proceed as before: Then begin with 3, the fourth Figure of the Multiplier, and ſay, 3 times 7 is 21, carry 2, and ſay, 3 times 3 is 9, and 2 I carry is 11; ſet down 1, and carry 1, and proceed as before: Then begin with 2, the fifth Figure, and ſay, 2 times 3 is 6, for which I carry 1, and ſay, 2 times 1 is 2, and 1 I carry is 3; ſet down 3; and 2 times 5 is 10; ſet down 0, and carry 1, and proceed as before: Then begin with 4, the laſt Figure of the Multiplier, and ſay, 4 times 1 is 4, for which I carry nothing, becauſe it is leſs than 5; then ſay, 4 times 5 is 10; ſet down 0, and carry 1, and proceed thro' the reſt of the Figures of the Multiplicand, Then add all up together, and the Product is 6276.9520.

See the **Work**.

375.13758 the Multiplicand.

4237.61 the Multiplier reverſed.

37513758 the Product with 1.

22508255 the Product with 6 increaſed with 6 x 8.

2625963 the Product with 7 increaſed with 7 x 5.

112541 the Product with 3 increaſed with 3 x 7.

7503 the Product with 2 increaſed with 2 x 3.

1500 the Product with 4 increaſed with 0.

6276.9520 the Product required.

Let the ſame *Example* be repeated, and let only one Place in Decimals be pricked off.

375.13758 the Multiplicand.
4237.61 the Multiplier inverted.

37514 the Product by 1 with the Increaſe of 1 x 7.
22508 the Product with 6 increaſed with 6 x 3.
2626 the Product with 7 increaſed with 7 x 1.
113 the Product with 3 increaſed with 3 x 5.
7 the Product with 2 increaſed with 2 x 7.
1 the Increaſe only of 4 x 3.

6276.9 the Product is the ſame as before.

More Examples for Practice.

Multiply 395.3756 by .75642, and prick off four Places in Decimals.

395.3756 the Multiplicand.
24657. the Multiplier reverſed.

2767629 the Product by 7 increaſed with 7 x 6.
197688 the Product by 5 increaſed with 5 x 5.
23722 the Product by 6 increaſed with 6 x 7.
1581 the Product by 4 increaſed with 4 x 3.
79 the Product by 2 increaſed with 2 x 5.

299.0699 the Product required.

Let the ſame *Example* be repeated, and let there be only One Place of Decimals.

395.3756
24657.

1767 the Product by 7 increaſed with 7 x 3.
198 the Product by 5 increaſed with 5 x 5.
24 the Product by 6 increaſed with 6 x 9 + 6 x 5.
2 the Increaſe of 4 x 9 + 4 x 3.

299.1 the Product.

Characters and their Signification.

NOTE, That this Mark + ſignifies Addition; 8+5 that is, 8 more 5, or 8 added to 5; and 8+3+7 denotes theſe Numbers are to be added into one Sum.

This Mark — ſignifies Subraction; as 9—4 ſignifies that 4 is to be taken from 9.

This Mark × ſignifies Multiplication; as 7 × 5 ſignifies that 7 is to be multiplied into 5.

This Mark — ſignifies Diviſion; as 12÷4 ſignifies 12 is to be divided by 4.

This Mark = ſignifies Equality, or Equation, that is, when = is placed between Numbers or Quantities, it denotes them to be equal; as 7+5=12, that is, 7 more 5 is equal to 12; and 15—7=8, that is, 15 leſs by 7, is equal to 8, or ſubtract 7 from 15 and there remains 8.

This Mark : : is the Sign of Proportion, or the Golden Rule, it being always placed betwixt the two middle Terms or Numbers in Proportion, thus, 4 : 20 : : 6 : 30; to be thus read, as 4 is to 20, ſo is 6 to 30.

CHAP.

CHAP. VI.

Diviſion of DECIMALS.

DIVISION of Decimals is performed after the ſame Manner as Diviſion of whole Numbers; but to know the Value or Denomination of the Quotient is the only Difficulty; for the reſolving of which obſerve either of the following

RULES.

I. The firſt Figure in the Quotient muſt be of the ſame Denomination with that Figure in the Dividend which ſtands (or is to be ſuppoſed to ſtand) over the Unit's Place in the Diviſor, at the firſt ſeeking.

II. When the Work of Diviſion is ended, count how many Places of decimal Parts there are in the Dividend more than in the Diviſor, for that Exceſs is the Number of Places which muſt be ſeparated in the Quotient for Decimals: But if there be not ſo many Figures in the Quotient, as is the ſaid Exceſs, that Deficiency muſt be ſupplied with Cyphers in the Quotient, prefixed before the ſignificant Figures thereof, towards the Left Hand, with a Point before them; ſo ſhall you plainly diſcover the value of the Quotient.

Theſe following Directions ought alſo to be carefully obſerved.

If the Diviſor conſiſts of more Places than the Dividend, there muſt be a competent Number of Cyphers annexed to the Dividend, to make it conſiſt of as many (at leaſt) or more Places of Decimals than the Diviſor; for the Cyphers added muſt be reckoned as Decimals.

Conſider whether there be as many decimal Parts in the Dividend as there are in the Diviſor; if there be not, make them ſo many, or more, by annexing of Cyphers.

In dividing of whole or mixed Numbers, if there be a Remainder, you may bring down more Cyphers, and, by continuing your Diviſion, carry the Quotient to as many Places of Decimals as you pleaſe.

Theſe Things being conſidered, I ſhall proceed to the Practice of Diviſion of Decimals, which I ſhall endeavour to explain in as familiar and eaſy a Method as poſſible.

Example 1. Let 48 be divided by 144.

In this *Example* the Diviſor 144 is greater than the Dividend 48; therefore, according to the Directions above, I annex a competent Number of Cyphers, (*viz.* four) with a Point between them, and divide according to the uſual Way.

144)48.0000(.3333

480
480
480

48

But, firſt, in ſeeking how often 144 in 480, (the firſt three Figures of the Dividend) I find the Unit's Place of the Diviſor to fall under the firſt Place of Decimals; therefore the firſt Figure in the Quotient is in the firſt Place of Decimals: Or, by the ſecond Rule, there being four Places of Decimals in the Dividend, and none in the Diviſor, ſo the Exceſs of decimal Places in the Dividend, above that in the Diviſor, is four; ſo that when the Diviſion is ended, there muſt be four Places of Decimals in the Quotient.

See the WORK.

Example 2. Let 217.75 be divided by 65.

Firſt, In ſeeking how often 65 in 217, (the firſt three Figures of the Dividend) I find the Unit's Place of the Diviſor to fall under the Unit's Place of the Dividend; therefore the firſt Figure in the Quotient will be Units, and all the reſt Decimals Or, by the ſecond Rule, there being two Places of Decimals in the Dividend, and no Decimals in the Diviſor, therefore the Exceſs of Decimal Places in the Dividend, above the Diviſor, is two; ſo when the Diviſion is ended, ſeparate two Places in the Quotient, towards the Right Hand, by a Point.

See the WORK.

65)217.75(3.35
· · ·

227
325

· · ·

Example 3. Let 267.15975 be divided by 13.25.

13.25)267.15975(20.163
· · · · ·

2159
8347
3975

· · · ·

In this *Example*, 3, the Unit's Place of the Diviſor, falls under 6, the Ten's Place of the Dividend; therefore (by the firſt Rule) the firſt Figure in the Quotient is Tens: Or, by the ſecond Rule, the Exceſs of decimal Places in the Dividend, above the Diviſor, is three; there being five Places of Decimals in the Dividend, and but two in the Diviſor, ſo there muſt be three Places of Decimals in the Quotient.

Example 4. Let 15.675159 be divided by 375.89.

375.89)15.675159(.0417
· ·

63955
263669

546

In this *Example*, 5, the Unit's Place of the Diviſor, falls under 7, the ſecond Place of Decimals in the Dividend; therefore (by the firſt Rule) the firſt Figure in the Quotient is in the ſecond Place of Decimals, ſo that you muſt put a Cypher before the firſt Figure in the Quotient: And by the ſecond Rule, the Exceſs of Decimal Places in the Dividend, above the Number of Decimal Places in the Diviſor, is 4; for the Decimal Places in the Dividend is 6, and the Number of Places in the Diviſor but two; therefore there muſt be four Places of Decimals in the Quotient. But the Diviſion being finiſhed after the common Way, the Figures in the Quotient are but three; therefore you muſt prefix a Cypher before the ſignificant Figures.

Example 5. Let 72.1564 be divided by .1347.

```
.1347)72.1564(535.68
           . . :
       ———————
        4806
         7654
          9190
          11080
         ———————
             304
```

In this *Example*, the Diviſor being a Decimal, the firſt Figure thereof falls under the Ten's Place in the Dividend; therefore the Units (if there had been any) ſhould fall under the Hundred's Place in the Dividend, and ſo the firſt Figure in the Quotient is Hundreds: And, by the ſecond Rule, there being four Places of Decimals in the Dividend, and as many in the Diviſor, ſo the Exceſs is nothing; but in dividing, I put two Cyphers to the Remainders, and continue the Diviſion to two Places farther; ſo I have two Places of Decimals.

See the WORK.

Example 6. Let .125 be divided by .0457.

```
.0457).1250000(2.735
         ....
       ------
       914
       ------
       3360
       3199
       ------
       1610
       1371
       ------
       2390
       2285
       ------
        105
```

In this *Example*, the Unit's Place of the Diviſor (if there had been any) would fall under the Unit's Place of the Dividend; therefore the firſt Figure of the Quotient is Units: And, by the ſecond Rule, there being ſeven Places of Decimals in the Dividend, and but four Places in the Diviſor, ſo the Exceſs is three; therefore there muſt be three Places of Decimals in the Quotient.

I ſhall ſet down only the Work of ſome few *Examples* more, and ſo proceed to *Contracted Diviſion.*

```
.00456).0000059791(.00131
                ..
              ------
              1419
                511
              ------
                 55
```

Let 1 be divided by 282, *&c.*

282)1.0000000(.0035461 *ferè.*
.....

1540
1300
1720
280

.325).400000(1.2307
...

750
1000
2500

225

.042)495.00000(11785.71
. .....

75
330
360
240
300
60

18

Diviſion of DECIMALS *contracted.*

IN Diviſion of Decimals the common Way, when the Diviſor hath many Figures, and it is required to continue the Diviſion till the Value of the Remainder be but ſmall, the Operation will ſometimes be large and tedious, but may be excellently contracted by the following Method.

The RULE.

By the firſt Rule of this Chapter, (*pag.* 19.) find what is the Value of the firſt Figure in the Quotient; then, by knowing the firſt Figure's Denomination, you may have as many or as few Places of Decimals as you pleaſe, by taking as many of the Left Hand Figures of the Diviſor as you think convenient for the firſt Diviſor; and then take as many Figures of the Dividend as will anſwer them; and in dividing, omit one Figure of the Diviſor at each following Operation.

A few *Examples* will make it plain.

Example 1. Let 721.17562 be divided by 2.25743, and let there be three Places of Decimals in the Quotient.

```
2.25743)721.175|62(319.467
 .....  677229
        ------
         43946
         22574
        ------
         21372
         20317
        ------
          1055
           903
          ----
           152
           135
          ----
            17
            15
           ---
             2
```

In this *Example*, the Unit's Place of the Diviſor falls under the Hundred's Place in the Dividend; and it is required that three Places of Decimals be in the Quotient; ſo there muſt be ſix Places in all, that is, three Places of whole Numbers, and three Places of Decimals: Then, becauſe I can have the Diviſor in the firſt ſix Figures of the Dividend, I cut off the 62 with a Daſh of the Pen, as uſeleſs; then I ſeek how oft the Diviſor in the Dividend, and the Anſwer is three times; put 3 in the Quotient, and multiply and ſubtract as in common Diviſion, and the Remainder is 43946: Then prick off the 3 in the Diviſor, and ſeek how oft the remaining Figures may be had in 43946, the Remainder, which can be but once; put 1 in the Quotient, and multiply and ſubtract, and the next Remainder is 21372: Then prick off the 4 in the Diviſor, and ſeek how often the remaining Figures may be had in 21372, which will be 9 times; put 9 in the Quotient; multiply thus, ſaying, 9 times 4 is 36, for which I carry 4, (in reſpect of the 4 laſt pricked off) and 9 times 7 is 63, and 4 is 67; ſet down 7 and carry 6, and ſo proceed till the Diviſion be finiſhed, always reſpecting the Increaſe made from the Figures pricked off. Obſerve the Work, which will better inform you than many Words.

```
2.25743)721.17562(319.461
                 . . .
        677229|
        ------ --
         43946|6
         22574|3
        ------ --
         21372|32
         20316|87
        ------ --
          1055|450
           902|972
        ------ ---
           152|4780
           135|4458
        ------ ----
            17|03220
            15|80201
        ------ -----
             1|23019
```

I have ſet down the Work of this laſt *Example* at large, according to the common Way, that thereby the Learner may ſee the Reaſon of the Rule, all the Figures on the Right Side the perpendicular Line being wholly omitted.

Example 2. Let 5171.59165 be divided by 8.758615, and let it be required that four Places of Decimals be pricked off in the Quotient.

```
8.758615)5171.5916|5(590.4577
  .......        .
          43793075
          --------
           7922841
           7882754
           -------
             40087
             35034
             -----
              5053
              4379
              ----
               674
               613
               ---
                61
                61
                --
                ..
```

In this *Example*, I can't have 8, the firſt Figure in the Diviſor, in 5, the firſt Figure of the Dividend; ſo that the Unit's Place of the Diviſor falls under the Hundred's Place of the Dividend; ſo that there will be ſeven Figures in the Quotient, that is, three of whole Numbers, and four of Decimals; therefore there muſt be ſeven Figures in the Diviſor, (becauſe the Number of Places in the Diviſor and Quotient will be equal) and there muſt be eight Places in the Dividend, ſo that I cut off the Figure 5 with a Daſh, as uſeleſs. Thus having proportioned the Dividend to the Diviſor, and both to the Number of Places or Figures deſired in the Quotient, I proceed to divide as before, ſaying, how often 8 in 51, which will be 5 times; put 5 in the Quotient, and multiply and ſubtract, and the Remainder is 7922841: Then I prick off the firſt Figure in the Diviſor, 5, and ſeek how often the remaining Figures of the Diviſor in the aforeſaid Remainder, which I find 9 times; put 9 in the Quotient, and multiply thereby, ſaying, 9 times 5 (the Figure pricked off) is 45, for which I carry 5, and ſay, 9 times 1 is 9, and 5 I carry is 14; ſet down 4, and carry 1, and proceed to multiply the reſt of the Figures, and ſubtract, and the Remainder will be 40087:

Then prick off the Figure 1, and ſeek how often 87586 in the Remainder 40087, the Anſwer will be 0; ſo put 0 in the Quotient, and prick off the Figure 6, and ſeek how often 8758 in 40087, which will be 4 times; put 4 in the Quotient, and multiply, ſaying, 4 times 6 (the Figure laſt prick'd off) is 24, for which I carry 2, and ſay, 4 times 8 is 32, and 2 I carry is 34; ſet down 4 and carry 3; multiply the reſt of the Figures, and ſubtract as before; and ſo proceed after the ſame Manner, until all the Figures of the Diviſor be prick'd off to the laſt Figure.

See the Work.

Example 3. Let 25.1367 be divided by 217.3543, and let there be five Places of Decimals in the Quotient.

In this *Example*, 7, the Unit's Place of the Diviſor, falls under 1, the firſt Place of Decimals, therefore the firſt Figure of the Quotient is in the firſt Place of Decimals, ſo the Quotient will be all Decimals: Then, becauſe the Quotient Figures and the Figures of the Diviſor will be of an equal Number daſh off the 43 in the Diviſor, and the 7 in the Dividend, as uſeleſs, and divide as before.

```
217.35|43)25.136|7(.11564
           21735
           -----
            3401
            2174
            ----
            1227
            1087
            ----
             140
             130
             ---
              10
               8
              --
               2
```

Altho' I have hitherto given Directions for proportioning the Diviſor and Dividend, ſo as to bring into the Quotient what Number of Decimals you pleaſe, yet there is no abſolute Neceſſity for it; but you may carry on your Diviſion to what Degree you pleaſe, before you begin to prick off the Figures of the Diviſor, in order to contract the Work, as in the following *Examples*, where it is not required to prick off any determinate Number of Decimals, but it may be done according to Diſcretion.

```
2.756756)7414.76717(2689.67118
 • •••••         •••
        5513512
        -------
        19012551
        16540536
        --------
        24720157
        22054048
        --------
         2666109
         2481080
         -------
          185029
          165405
          ------
           19624
           19297
           -----
             327
             276
             ---
              51
              28
              --
              23
              22
              --
               1
```

```
12.34254)514.75498(41.705757
           4937016
           -------
           2105338
           1234254
           -------
            871084
            863978
            ------
              7106
              6171
              ----
               935
               864
               ---
                71
                68
                --
                 9
                 8
                --
                 1
```

CHAP. VII.

Extraction of the SQUARE-ROOT.

If a Square *Number be given,*

TO find the Root thereof; that is, to find out ſuch a Number, as being multiplied into itſelf, the Product ſhall be equal to the Number given, ſuch Operation is called, *The Extraction of the* SQUARE-ROOT; which to do, obſerve the following Directions.

1ſt, You muſt point your given Number, that is, make a Point or Prick over the Unit's Place, another upon the Hundred's, and ſo upon every ſecond Figure throughout.

2dly, Then ſeek the greateſt ſquare Number in the firſt Point towards the Left Hand, placing the ſquare Number under the firſt Point, and the Root thereof in the Quotient, and ſubtract the ſaid ſquare Number from the firſt Point, and to the Remainder bring down the next Point, and call that the Reſolvend.

3dly, Then double the Quotient, and place it for a Diviſor, on the Left Hand of the Reſolvend, and ſeek how often the Diviſor is contained in the Reſolvend, (reſerving always the Unit's Place, and put the Anſwer in the Quotient, and alſo on the Right Hand Side of the Diviſor; then multiply by the Figure laſt put in the Quotient, and ſubtract the Product from the Reſolvend, (as in common Diviſion) and bring down the next Point to the Remainder, (if there be any more) and proceed as before.

A TABLE *of Squares and Cubes, and their Roots.*

Root	1	2	3	4	5	6	7	8	9
Squa.	1	4	9	16	25	36	49	64	1
Cube	1	8	27	64	125	216	343	512	729

Example 1. Let 4489 be a Number given, and let the ſquare Root thereof be required.

```
      ·  ·
      4489(67
      36
     ————
  127)889 Reſolvend
      889 Product
     ————
      ...
```

Firſt, point the given Number, as before directed; then (by the little Table aforegoing) ſeek the greateſt ſquare Number in 44, (the firſt Point to the Left Hand) which you will find to be 36, and 6 the Root; put 36 under 44, and 6 in the Quotient, and ſubtract 36 from 44, and there remains 8: Then to that 8 bring down the other Point 89, placing it on the Right Hand, ſo it makes 889 for a Reſolvend; then double the Quotient 6, and it makes 12, which place on the Left Hand for a Diviſor, and ſeek how often 12 in 88 (reſerving the Unit's Place) the Anſwer is 7 times, which put in the Quotient; and alſo on the Right Hand Side of the Diviſor, and multiply 127 by 7, (as in common Diviſion) and the Product is 889, which ſubtracted from the Reſolvend, there remains nothing; ſo is your Work finiſhed,—and the ſquare Root of 4489 is 67; which Root, if you multiply by itſelf, that is, 67 by 67, the Product will be 4489, equal to the given ſquare Number, and proves the Work to be right.

Example 2. Let 106929 be a Number given, and let the ſquare Root thereof be required.

```
        .  .  .
      106929(327
       9
      ----
   62)169 Reſolvend.
      124 Product.
      ----
  647)4529 Reſolvend.
      4529 Product.
      ----
      ....
```

Firſt, point your given Number, as before directed, putting a Point upon the Units, Hundreds, and Tens of Thouſands; then ſeek what is the greateſt ſquare Number in 10 (the firſt Point) which by the little Table you will find to be 9, and 3 the Root thereof; put 9 under 10, and 3 in the Quotient; then ſubtract 9 out of 10, and there remains 1, to which bring down 69, the next Point, and it makes 169 for the Reſolvend; then double the Quotient 3, and it makes 6, which place on the Left Hand of the Reſolvend for a Diviſor, and ſeek how often 6 in 16, the Anſwer is twice; put 2 in the Quotient, and alſo on the Right Hand of the Diviſor, making it 62: Then multiply 62 by the 2 you put in the Quotient, and the Product is 124; which ſubtract from the Reſolvend, and there remains 45; to which bring down 29, the next Point, and it makes 4529 for a new Reſolvend: Then double the Quotient 32, and it makes 64, which place on the Left Side the Reſolvend for a Diviſor, and ſeek how oft 64 in 452, which you will find 7 times; put 7 in the Quotient, and alſo on the Right Hand of the Diviſor, making it 647, which multiplied by 7 in the Quotient, it makes 4529; which ſubtracted from the Reſolvend, there remains nothing: So 327 is the ſquare Root of the given Number.

Example 3. Let 2268741 be a ſquare Number given, the Root whereof is required.

```
 ·  ·  ·
2268741)1506.23
1
25)126
   125
3006)18741
     18036
30122)70500
      60244
301243)1025600
        903729
Remains.121871
```

Having pointed the given Number, as before directed, ſeek what is the greateſt ſquare Number in the firſt Point 2, which is 1; put 1, the Square, under 2, and 1, the Root thereof, in the Quotient; ſubtract 1 from 2, and there remains 1; to which bring down the next Point 26, and ſet on the Right Hand, making it 126; double the 1 in the Quotient, which makes 2; ſet 2 on the Left Hand for a Diviſor, and ask how often 2 in 12, which will be five times; put 5 in the Quotient, and alſo on the Right Hand of the Diviſor, making it 25; multiply (as in common Diviſion) 25 by 5, and ſubtract the Product, 125, from 126, and there remains 1:

1: Bring down the next Point, 87, and it makes 187 for a new Reſolvend, and double the 15 in the Quotient, it makes 30 for a new Diviſor: Then ſeek how often 30 in 18, which you can't have; ſo that you muſt put 0 in the Quotient, and alſo on the Right Hand of the Diviſor, and bring down the next Point, and it makes 18741 for another new Reſolvend: Then ſeek how often 300 in 1874, which will be 6 times; put 6 in the Quotient, and alſo on the Right Hand of the Diviſor, multiply and ſubtract, and the Remainder will be 705. Now, if you have a mind to find the Value of the Remainder, you may annex Cyphers, by two at a time to the Remainders, and ſo proſecute the Work to what Number of decimal Parts you pleaſe; thus, to 705 annex two Cyphers, and it will make 70500, and the Quotient doubled, is 3012 for a Diviſor: Then ſeek how often 3012 in 7050 (rejecting the Unit's Place) which will be twice; put 2 in the Quotient, and alſo on the Right Hand of the Diviſor, and multiply and ſubtract as before, and the Remainder will be 10256; to which annex two Cyphers, and proceed as before, and you will get 3 in the Quotient next. So the ſquare Root of the given Number is 1506.23, which being ſquared or multiplied by itſelf, and the laſt Remainder added, will make the given Number as follows:

```
                        1506.23
                        1506.23
                        -------
                         451869
                        301246
                       903738
                      753115
                     150623
                     ------------
                     2268728.8129
The Remainder add---- -- 12.1871
                     ------------
              Proof  2268741.0000
```

Some

Some more Examples for Practice.

Example 1. 7596796(2756.128 Root,
4
47)359
329
545)3067
2725
5506)34296
33036
55122)126000
110244
551242)1575600
1102484
5512448)47311600
44099584
3212016

More

More Examples for Practice.

Example 2. 751417.5745(866.84 Root.
64

166)1114
996

1726)11817
10356

17328)146157
138624

173364)753345
693456

59889

If the given Number be a mixed Number, *viz.* conſiſting of a whole Number and a Decimal together, make the Number of decimal Places even, that is, 2, 4, 6, 8, *&c.* that ſo there may a Point fall upon the Unit's Place of the whole Numbers, as in this laſt *Example*, and in that following.

Example 3. Let 656714.37512 be given, to find the ſquare Root.

```
          . . . . . .
          656714.375120(810.379 Root.
          64
          ----
    161)167
        161
        -------
  16203)61437
         48609
         -------
 162067)1282851
        1134469
        --------
1620749)14838220
        14586741
        --------
   Remains 251479
        --------
```

In this *Example* there are five Places of Decimals; therefore put a Cypher to it, to make it even; that ſo there may a Point fall upon 4, the Unit's Place.

To find the Square Root of a Fraction.

If it be a decimal Fraction, the Work differs nothing from the *Examples* foregoing, only you muſt be mindful to point your given Number aright; for (as was before directed) the Number of Places muſt always be made even, and then begin to point at the Right Hand, as in whole Numbers.

If it be a Vulgar Fraction, it muſt be reduced to a Decimal, by the firſt Rule of the ſecond Chapter.

I ſhall give an *Example* or two in each Caſe, and ſo conclude this Chapter.

Let .125 be a decimal Fraction given, whoſe ſquare Root is required; and let it be required to have four Places of Decimals in the Root.

```
  .  .  .  .
.12500000(.3535
  9
65)350
   325
703)2500
    2109
7065)39100
      35325
       3775
```

In this *Example* there muſt be five Cyphers annexed, becauſe two Places in the Square make but one in the Root.

Let the Square Root of .00715 be required.

```
 .007150(.084
   64
164)750
    656
     94
```

In this a Cypher is added to make the Places even.

Let $\frac{7}{8}$ be a Vulgar Fraction given, whoſe Square Root is requir'd.

```
8)7000             .  .  .  .
  64             (.87500000(.9354
  ——               81
   60              ——
   56          183)650
  ——               549
   40              ——
   40         1865)10100
  ——                9325
   ..               ————
             18704)77500
                   74816
                   ————
                    2684
```

Reduce this $\frac{7}{8}$ to a Decimal, it makes .875, to which annex Cyphers, and extract the Square Root, as if it was a whole Number. So the Root is .9354.

Let $\frac{3}{960}$ be a Vulgar Fraction, whoſe Square Root is required.

```
                               .  .  .  .  Root.
96|0)3.0000000|0            (.00312500(.0559
       . . . :                    25
     288                     ——————
    ————                     105)625
     120                         525
      96                       ————
    ————                    1109)10000
     240                          9981
     192                         —————
    ————                             19
     480
     480
    ————
      ..
```

In extracting the Root of this, becauſe the firſt Point conſiſts of Cyphers, there muſt be a Cypher put firſt in the Quotient.

To prove this Rule, ſquare the Root, and to the Product add the Remainder, as was before directed. To ſquare a Number, is to multiply it by itſelf: and to cube it is to multiply the Square of the Number by the Number itſelf.

* * *

CHAP. VIII.

Extraction of the CUBE ROOT.

TO extract the CUBE ROOT, is nothing elſe but to find ſuch a Number, as being firſt multiplied into itſelf, and then into that Product, produceth the given Number; which to perform, obſerve theſe following Directions.

1ſt, You muſt point your given Number, beginning with the Unit's Place, and make a Point or Dot over every third Figure towards the Left Hand.

2dly, Seek the greateſt Cube Number in the firſt Point, towards the Left Hand, putting the Root thereof in the Quotient, and the ſaid Cube Number under the firſt Point, and ſubtract it therefrom, and to the Remainder bring down the next Point, and call that the Reſolvend.

3dly, Triple the Quotient, and place it under the Reſolvend, the Unit's Place of this under the Ten's Place of the Reſolvend; and call this the Triple Quotient.

4thly, Square the Quotient, and triple the Square, and place it under the triple Quotient; the Units of this under the Ten's Place of the triple Quotient, and call this the triple Square.

5thly, Add theſe two together, in the ſame Order as they ſtand, and the Sum ſhall be the Diviſor.

6thly, Seek how often the Diviſor is contained in the Reſolvend, rejecting the Unit's Place of the Reſolvend (as in the Square Root) and put the Anſwer in the Quotient.

7thly, Cube the Figure laſt put in the Quotient, and put the Unit's Place thereof under the Unit's Place of the Reſolvend.

8thly, Multiply the Square of the Figure laſt put in the Quotient, into the triple Quotient, and place the Product under the laſt, one Place more to the Left Hand.

9thly, Multiply the triple Square by the Figure laſt put in the Quotient, and place it under the laſt, one Place more to the Left Hand.

10thly, Add the three laſt Numbers together, in the ſame Order as they ſtand, and call that the Subtrahend.

Laſtly, Subtract the Subtrahend from the Reſolvend, and if there be another Point, bring it down in the Remainder, and call that a new Reſolvend, and proceed in all Reſpects as before, from the Beginning of the third Step to the End of this laſt.

Example 1. Let 314432 be a cubick Number, whoſe Root is required.

314432(68 Root.
216

98432 Reſolvend.

18 Triple Quotient of 6.
108 Triple Square of the Quotient 6.

1098 Diviſor.

512 Cube of 8, the laſt Figure of the Root.
1152 The Square of 8, by the triple Quotient.
864 The triple Square of the Quotient 6 by 8.

98432 The Subtrahend.

. . . . .

After you have pointed the given Number, ſeek what is the greateſt Cube Number in 314, the firſt Point, which, by the former little Table (*Page* 34) you will find to be 216, which is the neareſt that is leſs than 314, and its Root is 6; which put in the Quotient, and 216 under 314, and ſubtract it therefrom, and there remains 98, to which bring down the next Point, 432, and annex to 98, ſo will it make 98432 for the Reſolvend: Then triple the Quotient 6, it makes 18, which write down, the Unit's Place, 8, under 3, the Ten's Place of the Reſolvend: Then ſquare the Quotient 6, and triple that Square, and it makes 108, which write under the triple Quotient, one Place on the Left Hand; then add thoſe two Numbers together, and they make 1098 for the Diviſor: Then ſeek how often the Diviſor is contained in the Reſolvend (rejecting the Unit's Place thereof) that is, how often 1098 in 9843, which is eight times; put 8 in the Quotient, and the Cube thereof below the Diviſor, the Unit's Place under the Unit's Place of the Reſolvend: Then ſquare the 8 laſt put in the Quotient, and multiply 64, the Square thereof, by the triple Quotient 18, the Product is 1152; ſet this under the Cube of 8, the Units of this under the Tens of that.

Then multiply the triple Square of the Quotient by 8, the Figure laſt put in the Quotient, the Product is 864; ſet this down under the laſt Product, a Place more to the Left Hand: Then draw a Line under thoſe three, and add them together, and the Sum is 98432, which is called the Subtrahend; which being ſubtracted from the Reſolvend, the Remainder is nothing. Which ſhews the Number to be a true Cubick Number, whoſe Root is 68; that is, if 68 be cubed it will make 314432.

For, if 68 be multiplied by 68, the Product will be 4624, and this Product multiplied again by 68, the laſt Product is 314432; which ſhews the Work to be right.

```
                 68
                 68
                ----
                544
               408
               ----
The Work.      4624
                 68
              -----
              36992
             27744
             ------
The Proof    314432
             ------
```

Example 2. Let the Cube Root of 5735339 be required.

After you have pointed the given Number, ſeek what is the greateſt Cube Number in 5, the firſt Point, which (by the little Table, *Page* 34) you will find to be 1; which place under 5, and 1, the Root thereof, in the Quotient, and ſubtract 1 from 5, and there remains 4; to which bring down the next Point, it makes 4735 for the Reſolvend: Then triple the 1, and it makes 3, and the Square of 1 is 1, and the Triple thereof is 3; which ſet one under another, in their Order, and added makes 33 for the Diviſor: Seek how often the Diviſor in the Reſolvend, and proceed as in the laſt *Example.*

5735339

. . .
5735339(179 Root.
1

4735 Reſolvend.

3 Triple of the Quotient 1, the firſt Figure.
3 The triple Square of the Quotient 1.

33 The Diviſor.

343 The Cube of 7, the ſecond Figure of the Root.
147 The Square of 7, multip. in the triple Quotient 3.
21 The triple Square of the Quotient, multip. by 7.

8913 The Subtrahend.

822339 The new Reſolvend.

51 The Triple of the Quotient 17, the two firſt Figures.
867 The triple Square of the Quotient 17.

8721 Diviſor.

729 The Cube of 9, the laſt Figure of the Root.
4131 The Square of 9, multip. by the triple Quotient 51.
7803 The triple Square of the Quotient 867 by 9.

812339 The Subtrahend.

......

In this *Example*, 33, the firſt Diviſor, ſeems to be contained more than ſeven times in 4735, the Reſolvend; but if you work with 9, or 8, you will find that the Subtrahend will be greater than the Reſolvend.

Some

Some more Examples for Practice.

. . .

32461759(319 The Root.
27

5461 Reſolvend.

9 The Triple of 3.
27 The triple Square of 3.

279 The Diviſor.

1 The Cube of 1, the ſecond Figure.
9 The triple Quotient by the Square of 1.
27 The triple Square, multiplied by 1, the ſecond Figure.

2791 The Subtrahend.

2670759 A new Reſolvend.

93 The Triple of 31.
2883 The triple Square of 31.

28923 The Diviſor.

729 The Cube of 9, the laſt Figure.
7533 The Square of 9, by 93, the triple Quotient.
25947 The triple Square 2883 by 9.

2670759 The Subtrahend.

• • • • • • •

84604519(439 The Root.
64

20604 Reſolvend.

12 The Triple of 4.
48 The triple Square of 4.

492 The Divisor.

27 The Cube of 3.
108 The Square of 3 by the triple Quotient.
144 The triple Square by 3.

15507 The Subtrahend.

5097 59 Reſolvend.

129 The Triple of 43.
5547 The triple Square of 43.

55599 The Diviſor.

729 The Cube of 9.
10449 The Square of 9 by 129.
49923 The triple Square by 9.

5097519 The Subtrahend.

....

25266

259697989(638
216

43697 Resolvend.

18 The Triple of 6.
108 The triple Square of 6.

1098 The Divisor.

27 The Cube of 3, the ſecond Figure.
162 The Square of 3 by 18.
324 The triple Square 108 by 3.

34047 The Subtrahend.

9650989 Resolvend.

189 The Triple of 63.
11907 The triple Square of 63.

119259 The Divisor.

512 The Cube of 8.
12096 The Square of 8 by 189.
95256 The triple Square 11907 by 8.

9647072 The Subtrahend.

3917 The Remainder.

22069810125(2805
8

14069 Reſolvend.

6 The Triple of 2.
12 The triple Square of 2.

126 The Diviſor.

512 The Cube of 8.
384 The Square of 8 by 6.
96 The triple Square by 8.

13952 The Subtrahend.

117810125 New Reſolvend.

84 The Triple of 28.
2352 The triple Square of 28.

23604 Diviſor.

840 The Triple of 280.
235200 The triple Squa. of 280.

2352840 New Diviſor.

125 Cube of 5.
21000 Square of 5 by 840.
1176000 Triple Square by 5.

117810125 Subtrahend.

In this *Example*, 13952, being ſubtracted from the Reſolvend, 14069, the Remainder is 117, to which bring down 810, the third Point, and it makes 117810, for a new Reſolvend, and the next Diviſor is 23604, which you cannot have in the ſaid Reſolvend (the Unit's Place being rejected) ſo you muſt put 0 in the Quotient, and ſeek a new Diviſor; (after you have brought down your laſt Point to the Reſolvend) which new Diviſor is 2352840, which you will find to be contained five times. So proceed to finiſh the reſt of the Work.

25917056(295.9
8

17917 The Reſolvend.

6 The Triple of 2.
12 The triple Square of 4.

126 The Diviſor.

729 The Cube of 9, the ſecond Figure.
486 The Square of 9 by 6.
108 The triple Square by 9.

16389 The Subtrahend.

1528056 The Reſolvend.

87 The Triple of 29.
2523 The triple Square of 29.

25317 The Diviſor.

125 The Cube of 5, the 3d Figure.
2175 The Square of 5 by 87,
12615 The triple Square by 5.

1283375 The Subtrahend.

244681000 The Reſolvend.

885 The Triple of 295.
261075 The triple Square of 295.

2611635 The Diviſor.

729 The Cube of 9, the laſt Fig.
71685 The Square of 9 by 885,
2349675 The triple Square by 9.

235685079 The Subtrahend.

8995921 The Remainder.

In this *Example* I annex three Cyphers to the Remainder, which makes the 3d Reſolvend, by which Means I bring one Place of Decimals. And ſo you may proceed to more decimal Places at Pleaſure, by annexing 3 Cyphers to the next Remainder, and carrying on the Work as before.

9375)

93759.575070(45.42
64

29759 The Reſolvend.

12 The Triple of 4, the firſt Figure.
48 The triple Square of 4.

492 The Diviſor.

125 The Cube of 5, the ſecond Figure.
300 The Square of 5 by 12, the triple Quotient.
240 The triple Square by 5.

27125 The Subtrahend.

2634575 The Reſolvend.

135 The Triple of 45.
6075 The triple Square of 45.

60885 The Diviſor.

64 The Cube of 4.
2160 The Square of 4 by 135.
24300 The triple Square by 4.

2451664 The Subtrahend.

182911070 The Reſolvend.

1362 The Triple of 45.4.
618348 The triple Square of 45.4.

6184842 The Diviſor.

8 The Cube of 2.
5448 The Square of 2 by 1362.
1236696 The triple Square by 2.

123724088 The Subtrahend.

59186982 The Remainder.

In extracting the Cube Root of a mixed Number, always obſerve to make the decimal Part to conſiſt of either three, ſix, nine, *&c.* Places; that is, always to conſiſt of even Points, as in the laſt *Example*, where the decimal Places were five, to which I annexed a Cypher, to make up ſix, and ſo I proceed to point it; and by that Means I have a Point falls upon the Unit's Place of whole Numbers; which you muſt always obſerve.

To extract the Cube Root *out of a Fraction.*

This is the ſame to do as in whole Numbers, obſerve but the foregoing Directions for the true pointing thereof; for, as was before directed, the Decimal muſt always conſiſt of three, ſix, nine, *&c.* Places; and if it be not ſo it muſt be made ſo by annexing of Cyphers, as is above ſaid.

If the Cube Root of a Vulgar Fraction be required, you muſt firſt reduce it to a Decimal, and then extract the Root thereof.

Examples of each follow.

Example 1. Let the Cube Root of .401719179 be requir'd.

.401719179(.737 Root.
343

58719 Reſolvend.

21 Triple of 7.
147 Triple Square of 7.

1491 Diviſor.

27 Cube of 3.
189 Square of 3 by 21.
441 Triple Square by 3.

46017 Subtrahend.

12702179 Reſolvend.

219 Triple of 73.
15987 Triple Square of 73.

160089 Diviſor.

343 Cube of 7.
10731 Square of 7 by 219.
111909 Triple Square by 7.

11298553 Subtrahend.

1403626 Remainder.

Example 2. Let the Cube Root of .0001416 be required.

.   .   .
.0001416oo(.052 Root.
125

16600 Reſolvend.

15 The Triple of 5.
75 Triple Square of 5.

765 Diviſor.

8 Cube of 2.
60 The Square of 2 by 15.
150 Triple Square by 2.

15608 Subtrahend.

992 Remainder.

Example 3. Let $\frac{5}{276}$ be a Vulgar Fraction whoſe Cube Root is required.

By the firſt Rule of Chapter II. Reduce the Vulgar Fraction to a Decimal.

276)5.000000000(.018115942
. . . .

2240
320
440
1640
2600
1160
560

8

.018115942

.018115942(.262 Root.
8

10115 Reſolvend.

6 Triple of 2.
12 Triple Square of 2.

126 Diviſor.

216 Cube of 6.
216 Square of 6 by the Triple of 2.
72 The triple Square by 6.

9576 Subtrahend.

539942 Reſolvend.

78 Triple of 26.
2028 Triple Square of 26.

20358 Diviſor.

8 Cube of 2.
312 Square of 2 by 78.
4056 Triple Square 2028 by 2.

408728 Subtrahend.

131214 Remainder.

You may prove the Truth of the Work by cubing the Root found, as was ſhewed in the firſt *Example*; and if any Thing remains, add it to the ſaid Cube, and the Sum will be the given Number, if the Work is rightly performed.

I will ſhew the Proof of the fifth *Example*, (*Page* 48) the given Number being 259697989, whoſe Root is 638, it being a ſurd Number there remains 3917.

	638
	638
	5104
	1914
	3828
The Square	407044
	638
	3256352
	1221132
	2442264
The Cube	259694072
The Remainder add	3917
Proof equal to the given Number	259697989

* * *

CHAP. IX.

Multiplication of Feet, Inches and Parts.

IN the multiplying of Feet, Inches, *&c.* I ſhall endeavour to lay down ſuch eaſy and familiar Rules, as may eaſily be underſtood by the meaneſt Capacity.

Example 1. Let 7 Feet 9 Inches be multiplied by 3 Feet 6 Inches.

F.	I.	
7	9	
3	6	
23	3	Pts.
3	10	6
27	1	6

Firſt, multiply 9 Inches by 3, ſaying, 3 times 9 is 27 Inches, which make 2 Feet 3 Inches; ſet down 3 under Inches, and carry 2 to the Feet, ſaying, 3 times 7 is 21, and 2 that I carry make 23; ſet down 23 under the Feet.

Then begin with 6 Inches, ſaying, 6 times 9 is 54 Parts, which is 4 Inches and 6 Parts; ſet down 6 Parts, and carry 4, ſaying, 6 times 7 is 42, and 4 that I carry is 46 Inches, which is 3 Feet 10 Inches; which ſet down, and add all up together, and the Product is 27 Feet, 1 Inch, 6 Parts.

Example 2. Let 75 Feet 7 Inches, be multiplied by 9 Feet 8 Inches.

F.	I.	
75	7	
9	8	
680	3	
50	4	8
730	7	8

Firſt, multiply by 9 Feet, ſaying, 9 times 7 is 63, which is 5 Feet 3 Inches; ſet down 3 and carry 5, ſaying, 9 times 5 is 45, and 5 I carry is 50; ſet down 0, and carry 5, ſaying, 9 times 7 is 63, and 5 is 68, ſet down 68, and proceed to multiply by 8 Inches, ſaying, 8 times 7 is 56, the Twelves in 56 are 4 times, and 8 remains; ſet 8 a Place to the Right Hand, and carry 4; Then multiply 75 by 8, and the Product is 600, and 4 that I carry is 604, which divided by 12, the Quotient is 50 Feet, and 4 remains; ſet down 50 Feet, 4 Inches, and add all up together, and you will find the Product 730 Feet, 7 Inches, 8 Parts.

I will repeat the laſt *Example* again, and ſhew another Way to work it, which I think is better and more expeditious, when there are more Figures than one in the Feet: Thus,

F	I	P	
75	7		
9	8		
680	3		
25	2	4	1/3
25	2	4	1/3
730	7	8	

Multiply by 9 Feet firſt, as above directed; then, inſtead of multiplying by 8 Inches, let the 8 Inches be parted into ſuch aliquot or even Parts of a Foot as you find to be contained in that Figure; if you take ſuch Parts of the Multiplicand and add them to the former Product, the Sum will give the Anſwer: Thus, 8 Inches may be parted into 4 and 4, becauſe 4 is the third Part of 12: So, if you take the third Part of 75 Feet, 7 Inches, and ſet it down twice, and add all together, the Sum will be 730 Feet, 7 Inches, 8 Parts, the ſame as before: Thus, ſay, how often 3 in 7, which is twice, ſet down 2; then, becauſe twice 3 is 6, ſay, 6 out of 7 and there remains 1, for which you muſt add 10 to the 5, and it makes 15; then the Threes in 15 are 5 times, ſet down 5; and, becauſe 3 times 5 is 15, there is 0 remains. Then go to the 7 Inches, ſaying, the Threes in 7 are twice, ſet down 2 in the Inches; and, becauſe twice 3 is but 6, take 6 out of 7 and there remains 1 Inch, which is 12 Parts; then the Threes in 12 are 4 times, and 0 remains. So the third Part of 75 Feet 7 Inches, is 25 Feet, 2 Inches, 4 Parts; which ſet down again, and add all together, the Sum is 730 Feet, 7 Inches, 8 Parts, the ſame as before.

Example 3. Let 97 Feet, 8 Inches, be multiplied by 8 Feet, 9 Inches.

F.	I.	
97	8	$\frac{1}{2}$
8	9	$\frac{1}{4}$
781	4	
48	10	
24	5	
854	7	

Begin, firſt, to multiply by 8 Feet, ſaying, 8 times 8 is 64 Inches, that is, 5 Feet 4 Inches; ſet down 4 Inches, and carry 5, ſaying, 8 times 7 is 56, and 5 I carry is 61; ſet down 1, and carry 6, ſaying, 8 times 9 is 72, and 6 I carry is 78, which ſet down: Then, inſtead of multiplying by 9 Inches, take the aliquot Parts of 12, which 9 makes, which is 6 and 3, 6 Inches being half 12, and 3 the fourth Part; therefore take the half of 97 Feet, 8 Inches, which is 48 Feet, 10 Inches, and becauſe 3 is half 6, you may take the half of 48 Feet, 10 Inches, which is 24 Feet, 5 Inches; add all up together, and the Sum is 854 Feet, 7 Inches. See the Work, as above.

Example 4. Let 75 Feet, 9 Inches, be multiplied by 17 Feet 7 Inches.

F.	I.	
75	9	
17	7	
525		
75		
25	3	P.
18	11	3
8	6	
4	3	
1331	11	3

In this *Example*, becauſe there are more than 12 Feet in the Multiplier, therefore I firſt multiply the 75 by 17 Feet; then becauſe the aliquot Parts in 7 Inches are 4 and 3, that is, a Third and a Fourth, I take the third Part of 75 Feet, 9 Inches, which is 25 Feet, 3 Inches, and the fourth Part thereof is 18

Feet, 11 Inches, 3 Parts; then the aliquot Parts of 9 Inches are 6 and 3, that is, half and a fourth; therefore I take half 17 Feet, which is 8 Feet, 6 Inches, and the fourth Part is 4 Feet, 3 Inches, (not meddling with the 7 Inches, becauſe that was multiplied into the 9 before) then add all theſe together, and the Sum is 1331 Feet, 11 Inches, 3 Parts.

Example 5. Let 87 Feet, 5 Inches, be multiplied by 35 Feet, 8 Inches.

F.	I.		
87	5		
35	8		
435			
261		P.	
29	1	8	$\frac{1}{3}$
29	1	8	$\frac{1}{3}$
11	8	0	$\frac{1}{3}$
2	14	0	$\frac{1}{12}$
3117	10	4	

Work here as in the laſt *Example.* After you have multiplied the Feet, then take the aliquot Parts of 8 Inches, which is two Thirds; therefore take the third Part of 87 Feet 5 Inches, and ſet it down twice; thus, the third Part of 87 Feet 5 Inches, is 29 Feet, 1 Inch, 8 Parts; ſet this down twice: Then the aliquot Parts of 5 Inches are 4 and 1, that is, a third Part and a twelfth Part; therefore take a third Part of 35, which is 11 Feet, 8 Inches, and a twelfth Part of 35 is 2 Feet, 11 Inches; ſet all theſe one under another, and add them together, and the Sum is 3117 Feet, 10 Inches, 4 Parts.

Example 6. Let 259 Feet, 2 Inches, be multiplied by 48 Feet, 11 Inches.

F.	I.			
259	2			
48	11			
2072				
1036				
129	7	P.	$\frac{1}{2}$	
86	4	8	$\frac{1}{3}$	
21	7	2	$\frac{1}{12}$	
8	1	10	$\frac{1}{6}$	of 48 11.
12677	8	8		

Firſt, multiply the Feet; then take the aliquot Parts of 11, which will be 6, 4 and 1, that is, an Half, a Third, and a Twelfth; therefore take the Half of 259 Feet 2 Inches, which is 129 Feet 7 Inches, and a third Part is 86 Feet, 4 Inches, 8 Parts, and the twelfth Part of 259 Feet 2 Inches, is 21 Feet, 7 Inches, 2 Parts; or (becauſe 1 is the fourth Part of 4) you may more readily take the fourth Part of 86 Feet, 4 Inches, 8 Parts, which is alſo 21 Feet, 7 Inches, 2 Parts; then becauſe 2 Inches (in the Multiplicand) is one Sixth of 12, take the ſixth Part of the Feet, only, in the Multiplier, *viz.* 48, which quotes 8 Feet, and add all together, and the Sum is 11677 Feet, 6 Inches, 10 Parts. See the foregoing Work, corrected.

I ſhall ſet down only the working of ſome few *Examples* in Feet and Inches, and then proceed to multiply Feet, Inches and Parts, *&c.*

	F.	I.	P.		
	179	3			
	38	10			
	1432				
	537				
	89	7	6	$\frac{1}{2}$	
	59	9	0	$\frac{1}{3}$	F.
	9	6	0	$\frac{1}{4}$	of 38
Product	6960	10	6		

	F.	I.	P.		
	246	7			
	46	4			
	1476				
	984				
	82	2	4	$\frac{1}{3}$	F.
	15	4	0	$\frac{1}{3}$	of 46
	11	6	0	$\frac{1}{4}$	
Prod.	11425	0	4		

	F.	I.	P.		
	246	7			
	36	9			
	1476				
	738				
	123	3	6	$\frac{1}{2}$	
	61	7	9	$\frac{1}{4}$	F.
	12	0	0	$\frac{1}{3}$	of 36
	9	0	0	$\frac{1}{4}$	
Prod.	9061	11	3		

	F.	I.	P.		
	257	9			
	39	11			
	2313				
	771				
	128	10	6	$\frac{1}{2}$	
	85	11	0	$\frac{1}{3}$	
	21	5	9	$\frac{1}{12}$	F.
	19	6	0	$\frac{1}{2}$	of 39
	9	9	0	$\frac{1}{4}$	
Prod.	10288	6	3		

Example 11. Let 7 Feet, 5 Inches, 9 Parts, be multiplied by 3 Feet, 5 Inches, 3 Parts.

F.	I.	P.		
7	5	9		
3	5	3		
22	5	3	S.	
3	1	4	9	T.
	1	10	5	3
25	8	6	2	3

In this *Example*, I firſt begin with three Feet, and thereby multiply 7 Feet, 5 Inches, 9 Parts: Firſt I ſay, 3 times 9 is 27 Parts, that is, 2 Inches and 3 Parts; ſet down 3 under the Parts, and carry 2, ſaying, 3 times 5 is 15, and 2 I carry is 17, that is, 1 Foot 5 Inches; ſet down 5 Inches and carry 1, and ſay, 3 times 7 is 21, and 1 I carry is 22; ſet down 22 Feet. Then begin with 5 Inches, ſaying, 5 times 9 is 45, which is 45 Seconds, which make 3 Parts and 9 Seconds; ſet down 9 Seconds a Place towards the Right Hand, and carry 3 Parts, ſaying, 5 times 5 is 25, and 3 I carry is 28, which is 2 Inches and 4 Parts; ſet down 4 Parts, and carry 2, ſaying, 5 times 7 is 35, and 2 I carry is 37, which is 3 Feet 1 Inch; ſet down 3 Feet 1 Inch, and begin to multiply by 3 Parts, ſaying, 3 times 9 is 27 Thirds, that is, 2 Seconds and 3 Thirds; ſet down 3 Thirds, and carry 2, ſaying, 3 times 5 is 15, and 2 I carry is 17, that is, 1 Part and 5 Seconds; ſet down 5 Seconds, and carry 1, ſaying, 3 times 7 is 21, and 1 I carry is 22, which is 1 Inch and 10 Parts; which ſet down, and add all up, and the Product is 25 Feet, 8 Inches, 6 Parts, 2 Seconds, 3 Thirds.

NOTE, That in multiplying Feet, Inches, and Parts, *&c.* if Feet be multiplied by Feet, the Product is Feet; and Feet multiplied by Inches, the Product is Inches, and the twelfth Part is Feet; and Parts multiplied by Feet, the Product is Parts, and the twelfth Part thereof is Inches; Parts multiplied by Inches, the Product is Seconds, and the twelfth Part thereof is Parts; and Parts multiplied by Parts, the Product is Thirds, and the twelfth Part thereof is Seconds.

So that if you begin to multiply Parts by Feet in the firſt Row, and Parts by Inches in the ſecond Row, and Parts by Parts in the third Row, the firſt Figure in every Row will ſtand a Place more towards the Right Hand, as you may ſee in the laſt *Example*.

Example 12. Let 37 Feet, 7 Inches, 5 Parts, be multiplied by 4 Feet, 8 Inches, 6 Parts.

F.	I.	P.			
37	7	5			
4	8	6			
150	5	8	S.		
12	6	5	8	–	$\frac{1}{3}$
12	6	5	8	T.	$\frac{1}{3}$
1	6	9	8	6	$\frac{1}{2}$
177	1	5	0	6	

Firſt, I multiply by 4 Feet, ſaying, 4 times 5 is 20, which is 1 Inch, 8 Parts; ſet down 8, and carry 1, ſaying, 4 times 7 is 28, and 1 I carry is 29, which is 2 Feet, 5 Inches; ſet down 5 Inches, and carry 2, ſaying, 4 times 7 is 28, and 2 I carry is 30; ſet down 0, and carry 3, and ſay, 4 times 3 is 12, and 3 is 15; ſet down 15. Then I begin with 8 Inches; but becauſe the Feet in the Multiplicand are more than 12, it will be the beſt Way to work for the aliquot Parts of 8; ſo here I work for 4 Inches, and ſet that down twice, 4 being the third Part of 12; therefore take the third Part of 37 Feet, 7 Inches, 5 Parts, which is 12 Feet, 6 Inches, 5 Parts, 8 Seconds; ſet this down twice. Then begin with 6 Parts; but, inſtead of multiplying, take half 37 Feet, 7 Inches, 5 Parts (becauſe 6 is half 12) and ſet it a Place more to the Right Hand: Thus, the half of 37 Feet is 18, which I muſt count 18 Inches, becauſe the Multiplier is 6 Parts; ſo the half of 37 Feet, 7 Inches, 5 Parts, is 1 Foot, 6 Inches, 9 Parts, 8 Seconds, 6 Thirds; which ſet down, and add all up together, and the Sum is 177 Feet, 1 Inch, 5 Parts, 0 Seconds, 6 Thirds.

***Example* 13.** Let 311 Feet, 4 Inches, 7 Parts, be multiplied by 36 Feet, 7 Inches, 5 Parts.

F.	I.	P.	S.	T.	
311	4	7			
36	7	5			
1866					
933					
103	9	6	4		$\frac{1}{3}$
77	10	1	9		$\frac{1}{4}$
8	7	9	6	4	$\frac{1}{3}$
2	1	11	4	7	$\frac{1}{4}$
12	0	0	0	0	$\frac{1}{3}$
1	0	0	0	0	$\frac{1}{12}$
	9	0	0	0	$\frac{1}{3}$
11402	2	4	11	11	

In this *Example*, becauſe the Feet both in the Multiplier and Multiplicand are compound Numbers, I firſt multiply the Feet one by the other; then take the aliquot Parts of 7 Inches, which are 4 Inches and 3, that is, a third and a fourth Part; ſo take the third Part of 311 Feet, 4 Inches, 7 Parts, which is 103 Feet, 9 Inches, 6 Parts, 4 Seconds, and the fourth Part is 77 Feet, 10 Inches, 1 Part, 9 Seconds; ſet theſe down one under another, the Feet under the other Feet; then the aliquot Parts of 5 Parts are 4 and 1, that is, a third and twelfth Part; ſo the third Part of 311 Feet, 4 Inches, 7 Parts, is 103 Feet, 9 Inches, 6 Parts, 4 Seconds; but becauſe the Multiplier is Parts, it muſt be ſet a Place to the Right Hand, that is, the 103 muſt be Inches, which is 8 Feet, 7 Inches; therefore I ſet down 8 Feet, 7 Inches, 9 Parts, 6 Seconds, 4 Thirds.

Then, becauſe 1 Inch is a fourth Part of 4 Inches, therefore I take a fourth Part of 8 Feet, 7 Inches, 9 Parts, 6 Seconds, 4 Thirds, which is 2 Feet, 1 Inch, 11 Parts, 4 Seconds, 7 Thirds; which is the ſame as if I had taken a twelfth Part of 311 Feet, 4 Inches, 7 Parts. Then, for 4 Inches in the Multiplicand, inſtead of multiplying 36 Feet by it, take a third Part, becauſe 4 Inches is a third Part of 12; ſo the third Part of 36 is 12 Feet, and the aliquot Parts of 7 Parts are 4 and 3, that is, a Third and a Fourth; ſo the third Part of 36 is 12, which now is 12 Inches, that is, 1 Foot, and the fourth Part is 9 Inches; add all theſe together, and the Sum will be 11402 Feet, 2 Inches, 4 Parts, 11 Seconds, 11 Thirds.

Example 14. Let 8 Feet, 4 Inches, 3 Parts, 5 Seconds, 6 Thirds, be multiplied by 3 Feet, 3 Inches, 7 Parts, 8 Seconds, 2 Thirds.

	F.	I.	P.	S.	T.				
	8	4	3	5	6				
	3	3	7	8	2				
	25	0	10	4	6				
	2	1	0	10	4	6			
		4	10	6	0	2	6		
			5	6	10	3	8	0	
				1	4	8	6	11	0
Product	27	7	3	5	1	8	8	11	0

In this laſt *Example* there is no Difficulty, if you do but obſerve the former Directions, and ſet every Row a Place more to the Right Hand.

I ſhall only ſet down the Working of ſome few *Examples* more, and ſo conclude this Chapter.

F.	I.	P.		
321	7	3		
9	3	6		
2894	5	3	S.	
80	4	9	9	T.
13	4	9	7	6
2988	2	10	4	6

F.	I.	P.		
42	7	8		
7	3	6		
298	5	8		
10	7	11	S.	
1	9	3	10	
310	10	10	10	

F.	I.	P.		
124	7	9		
14	6	2		
496				
124			S.	
62	3	10	6	T.
1	8	9	3	6
7	0	0	0	0
1	2	0	0	0
	7	0	0	0
	3	6	0	0
1809	1	1	9	6

F.	I.	P.		
259	10	8		
18	5	4		
2072				
259			S.	
86	7	6	8	
21	7	10	8	T.
7	2	7	6	8
9	0	0	0	0
6	0	0	0	0
1	0	0	0	0
4793	6	0	10	8

```
  F.  I.  P.
 267   7  10
  25   9   7
────────────────
1335
534
 133   9  11  S.
  66  10  11   6
  11   1   9  11  T.
   1  10   3   7  10
  12   6   0   0   0
   2   1   0   0   0
   1   0   6   0   0
       8   1   0   0
────────────────
6905   0   7   0  10
```

```
   F.  I.  P.
  317   9   7
   37   5   9
────────────────
 2219
 951           S.
  105  11   2   4
   26   5   9   7  T.
   13   2  10   9   6
    6   7   5   4   9
   18   6   0   0   0
    9   3   0   0   0
    1   6   6   0   0
        3   1   0   0
────────────────
11910   9  11   1   3
```

The

The RULE at large.

Feet multiplied by	*Feet*	*produce*	*Feet.*
	Inches		*Inches.*
	Parts		*Parts.*
Inches multiplied by	*Feet*	*produce*	*Inches.*
	Inches		*Parts.*
	Parts		*Seconds.*
Parts multiplied by	*Feet*	*produce*	*Parts.*
	Inches		*Seconds.*
	Parts		*Thirds.*

N. B. *Twelve Thirds make one Second, Twelve Seconds make one Part, twelve Parts make one Inch, and twelve Inches make one Foot, in this Multiplication of Feet, Inches and Parts.*

THE

THE

Complete Meaſurer.

PART II.

CHAP. I.

Menſuration of SUPERFICIES.

Superficial Figures are all ſuch as have only Length and Breadth, not having any commenſurable Thickneſs.

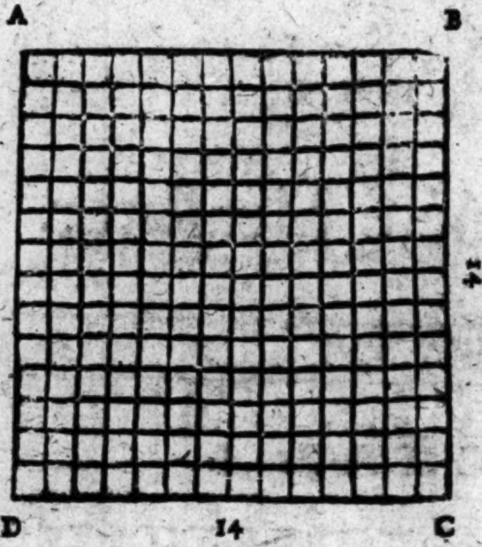
§ I. *Of a* SQUARE.

A SQUARE is a Geometrical Figure, having four equal Sides, and as many right (or ſquare) Angles. To find the ſuperficial Content thereof, this is

The RULE.

Multiply the Side into itſelf, and the Product is the Content.

Let ABCD be a Geometrical Square given; each Side being 14 Feet, Yards, Poles, or other Meaſure; multiply 14 by itſelf, and the Product is 196, which is the ſuperficial Content.



14
14

56
14

196 Product.

By Scale *and* Compaſſes.

Extend the Compaſſes from 1, in the Line of Numbers, to 14; the ſame Extent will reach from the ſame Point turned forward to 196.

DEMONSTRATION.

Let each Side of the given Square be divided into 14 equal Parts, and Lines drawn from one another, croſſing each other within the Square; ſo ſhall the whole great Square be divided into 196 little Squares, as you may ſee in the Figure above, equal to the Number of ſquare Feet, Yards, or Poles, or other Meaſures, by which the Side was meaſured.

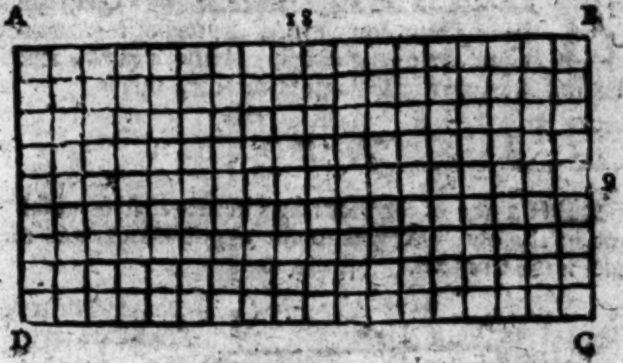
§ II.

§ II. *Of a* PARALLELOGRAM, *or* LONG SQUARE.

A Parallelogram is a Figure having four Sides, and as many right Angles, the oppoſite Sides thereof being equal and parallel. To find the ſuperficial Content thereof, this is

The RULE.

Multiply the Length by the Breadth, and the Product is the ſuperficial Content.



Length 18
Breadth 9

Product 162

Let ABCD be a long Square, the Length thereof 18 Feet, and the Breadth 9 Feet, which multiplied together, the Product is 162, the ſuperficial Content thereof.

By Scale *and* Compaſſes.

Extend the Compaſſes in the Line of Numbers from 1 to 9, the ſame Extent will reach from 18 down to 162, the ſquare Feet.

DEMON-

DEMONSTRATION.

If the Sides AB and CD, be each divided into 18 equal Parts, repreſenting 18 Feet, and the Lines AD and BC each divided into nine equal Parts, and Lines drawn from Point to Point, croſſing each other within the Figure; thoſe Lines will make thereby ſo many little Squares as there are ſquare Feet, *viz.* 162.

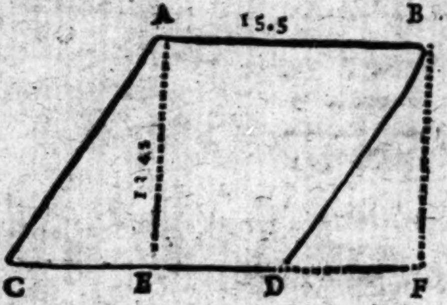
* * *

§ 3. *Of a* RHOMBUS.

A Rhombus is a Figure repreſenting a Quarry of Glaſs, having four equal Sides, the Oppoſites thereof being equal, two Angles being obtuſe, and two acute. To To find the ſuperficial Content thereof, this is

The RULE.

Multiply one of the Sides by a Perpendicular let fall from one of the obtuſe Angles to the oppoſite Side, and the Product is the Content.



Perpendicular 13.42
The Side 15.5

6710
6710
1342

Product 208.010

Let ABCD be a Rhombus given, whoſe Sides are each 15.5 Feet, and the Perpendicular EA is 13.42, which multiplied together, the Product is 208.01; which is the ſuperficial Content of the Rhombus, that is, 208 Feet, and one hundredth Part of a Foot.

By

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 13.42; that Extent will reach from 15.5, the ſame way to 208 Feet, the Content,

DEMONSTRATION.

Let CD be extended out to F, making DF equal to CE, and draw the Line BF; ſo ſhall the Triangle DBF be equal to the Triangle ACE: For DF and CE are equal, and BF is equal to AE, becauſe AB and CF are parallel. Therefore the Parallelogram ABEF it equal to the Rhombus ABCD.

§ IV. *Of a* RHOMBOIDES.

A Rhomboides is a Figure having four Sides, the oppoſite whereof are equal and parallel; and alſo four Angles, the oppoſite whereof are equal. To find the ſuperficial Content thereof, this is

The RULE.

Multiply one of the longeſt Sides thereof by the Perpendicular let fall from one of the obtuſe Angles to one of the longeſt Sides, and the Product is the Content.

```
  19.5
  10.2
  ----
   390
 1950
 ------
 198.90
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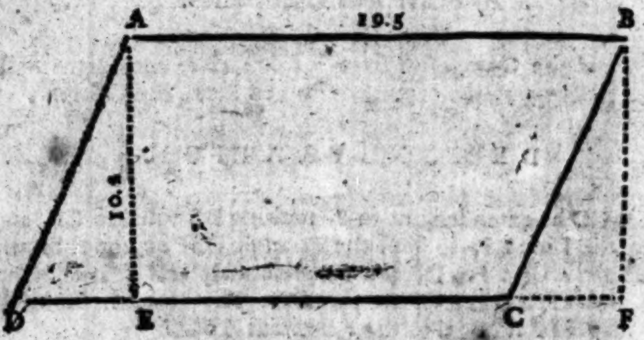
H Let



Let ABCD be a Rhomboides given, whoſe longeſt Sides AB or CD is 19.5 Feet, and the Perpendicular AE is 10.2; which multiplied together, the Product is 198.9, that is, 198 ſuperficial Feet, and 9 tenth Parts the Content.

DEMONSTRATION.

If DC be extended to F, making CF equal to DE, and a Line drawn from B to F; ſo will the Triangle CBF be equal to the Triangle ADE, and the Parallelogram AEFB be equal to the Rhomboides ABCD. Which was to be proved.

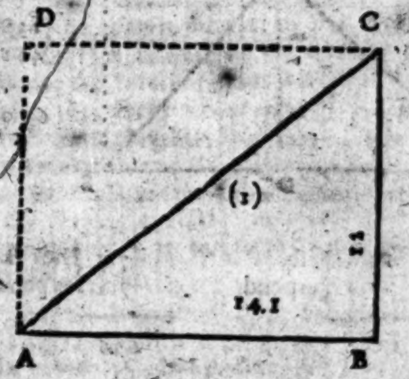
◆ ◆ ◆

§ V. Of a TRIANGLE.

A Triangle is a Figure having three Sides and three Angles. Triangles are right-angled or oblique-angled. Right-angled Triangles are ſuch as have one right Angle. Oblique-angled Triangles are ſuch as have their Angles either acute or obtuſe. An obtuſe Angle is greater than a right Angle, that is, it is more than 90 Degrees; and an acute Angle is leſs than a right Angle. To find the ſuperficial Content thereof, this is

The RULE.

Let the Triangle be of what kind ſoever, multiply the Baſe by half the Perpendicular, or half the Baſe by the whole Perpendicular; or multiply the whole Baſe by the whole Perpendicular, and take half the Product: Any of theſe three Ways will give the Content.



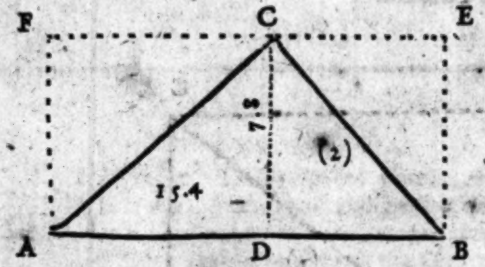
Let ABC be a right-angled Triangle, whoſe Baſe is 14.1 Feet, and the Perpendicular 12 Feet, multiply 14.1 by 6, half the Perpendicular, and the Product is 84.6 Feet, the Content. Or multiply 14.1 by 12, the Product is 169.2; the half thereof is 84.6, the ſame as before.

14.1 Baſe
6 Half Perpendicular.
84.6 Product.

14.1 Baſe.
12 Perpendicular.
169.2 Product.
84.6 Half.

By Scale *and* Compaſſes.

Extend the Compaſſes from 2 to 14.1, that Extent will reach the ſame Way from 12 to 84.6 Feet, the Content.



15.4 Baſe.
3.9 halfperpen

1386
462

60.06 Product.

15.4 Baſe.	7.7 Half Baſe.
7.8 Perpend.	7.8 Perpend.
1232	616
1078	539
120.12	60.06 Product.
60.06 Half.	

Let ABC (*Fig.* 2.) be an oblique-angled Triangle given, whoſe Baſe is 15.4, and the Perpendicular 7.8; if 15.4 be multiplied by 3.9, (half the Perpendicular) the Product will be 60.06 for the Area or ſuperficial Content: Or, if the Perpendicular 7.8 be multiplied into half the Baſe 7.7, the Product will be 60.06, as before: Or, if 15.4, the Baſe, be multiplied by the whole Perpendicular 7.8, the Product will be 120.12, which is the double Area; the Half thereof is 60.06 Feet, as before.

See the Work.

By Scale *and* Compaſſes.

Extend the Compaſſes from 2 to 15.4, that Extent will reach from 7.8 to 60 Feet, the Content.

DEMONSTRATION.

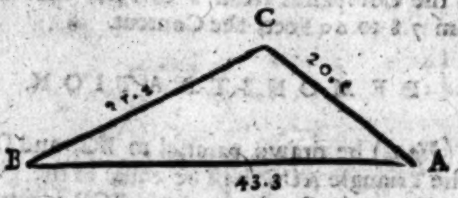
If AD (*Fig* 1.) be drawn parallel to BC, and DC parallel to AB; the Triangle ADC ſhall be equal to the given Triangle ABC. Hence the Parallelogram ABCD is double to the given Triangle; therefore half the Area of the Parallelogram is the Area of the Triangle. In *Fig*. 2. the Parallelogram ABEF is alſo double to the Triangle ABC; for the Triangle ACF is equal to the Triangle ACD, and the Triangle BCE is equal to the Triangle BCD; therefore the Area of the Parallelogram is double to the Area of the given Triangle. Which was to be proved.

To find the Area of any plain Triangle, by having the three Sides given, without the Help of a Perpendicular.

The RULE.

Add the three Sides together, and take half that Sum; then ſubtract each Side ſeverally from that half Sum: Which done, multiply that half Sum and the three Differences continually, and out of the laſt Product extract the ſquare Root, which ſquare Root ſhall be the Area of the Triangle ſought.

Example. Let ABC be a Triangle, whoſe three Sides are as followeth, *viz.* AB 43.3; AC 20.5, and BC 31.2; the Area is required.



Sides	Differences
43.3	4.2
31.2	16.3
20.5	27.0

Sum 95.0

Half 47.5

```
Area 296.31      47.5 The half Sum.
                   27 Difference.

                 3325
                 950

               1282.5 Product.
                 16.3 Difference.

               38475
              76950
             12825

             20904.75 Product.
                  4.2 Difference.

             4180950
            8361900

                 .  .  .  .  .
               87799.9500(296.31
               4

              49)477
                 441

             586)3699
                 3516

           5923)18395
                17769

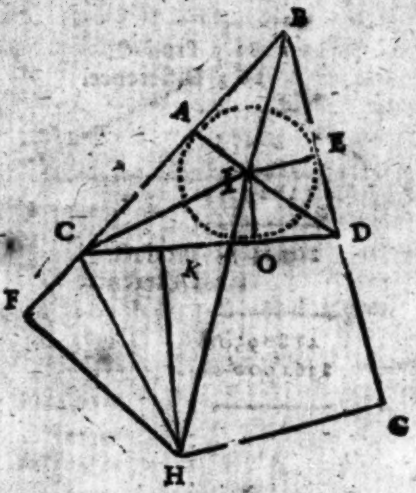
          59261)62600
                59261

                3339 Remains.
```

DEMON.

DEMONSTRATION.

In the Triangle BCD, I ſay, if from the half Sum of the Sides you ſubtract each particular Side, and multiply the half Sum and the three Differences together continually, the ſquare Root of the Product ſhall be the Area of the Triangle.



Firſt, by the Lines BI, CI and DI, biſect the three Angles, which Lines will all meet in the Point I; by which Lines the given Triangle is divided into three new Triangles, CBI, DCI, and BDI; the Perpendiculars of which new Triangles are the Lines AI, EI and OI, being all equal to one another, becauſe the Point I is the Center of the inſcribed Circle (by *Euclid, Lib.* 4. *Prop.* 4.) Wherefore to the Side BC join CF equal to DE or DO; ſo ſhall BF be equal to half the Sum of the Sides, *viz.* $= \frac{1}{2}BC + \frac{1}{2}BD + \frac{1}{2}CD$.

And $BA = BF - CD$; for $CA = CO$ and $OD = CF$, therefore $CD = AF$, and $AC = BF - BD$; for $BE = BA$, and $ED = CF$; therefore $BD = BA + CF$, and $CF = BF - BC$.

Then make $CK = CE$, and draw the Perpendiculars FH, GH and KH, and extend BI to H; becauſe the Angles $FCK + FHK$ are equal to two right Angles, (for the Angles F and K are right Angles) equal alſo to $FCK + ACO$ (by *Euclid*, 1. 13.)

And the Angles $ACO + AIO$ are equal to two right Angles; therefore the Quadrangles $FCKH$ and $AIOC$ are alike; and the Triangles CFH and AIC are alſo ſimilar. And the Triangles BAI and BFH are likewiſe ſimilar.

From this Explanation, I ſay, the Square of the Area of the given Triangle will be $BFq \times IAq = BF \times BA \times CA \times CF$. In Words:

The Square of BF (the half Sum of the Sides) multiplied into the Square of IA ($= IF = IO$) will be equal to the ſaid half Sum multiplied into all the three Differences.

For $IA : BA :: FH : BF$, and $IA : CF :: AC : FH$; becauſe the Triangles are ſimilar. By *Euclid, Lib. 6. Prop. 4.*

Wherefore multiplying the Extremes and Means in both, it will be $IAq \times BF \times FH = BA \times CA \times CF \times FH$; but FH being on both Sides of the Equation, it may be rejected; and then multiply each Part by BF, it will be $BFq \times IAq = BF \times BA \times CA \times CF$. Which was to be demonſtrated.

§ VI. *Of a* TRAPEZIUM.

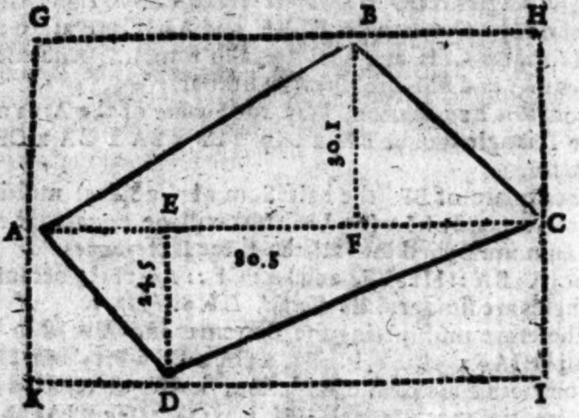
A Trapezium is a Figure having four unequal Sides and oblique Angles To find the Area or ſuperficial Content thereof, this is:

The RULE.

Add the two Perpendiculars together, and take half the Sum, and multiply that half Sum by the Diagonal, or multiply the whole Sum by half the Diagonal, the Product is the Area. Or you may find the Areas of the two Triangles ABC and ACD, (by Section V.) and add thoſe two Areas together, the Sum ſhall be the Area of the Trapezium.

Or, *laſtly*, multiply the whole Diagonal by the Sum of the Perpendiculars, and take half the Product for the Area of the Trapezium.

$BF = 30.1$



BF=30.1
DE=24.5

Sum 54.6

Half 27.3
AC=80.5

1365
2184

Area 2197.65

Let ABCD be a Trapezium given, the Diagonal whereof is 80.5, and the Perpendicular BF 30.1, and the Perpendicular DE 24.5; theſe two added together the Sum is 54.6, the half thereof is 27.3, which multiplied by the Diagonal 80.5, the Product is 2197.65, which is he Area of the Trapezium; or if 40.25, half the Diagonal, be multiplied by 54.6, the whole Sum of the Perpendiculars, the Product is 2197.65, the ſame as before.

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 54.6; that Extent will reach from 80.5 to 2197.65, the Area.

DEMONSTRATION.

This Figure, ABCD, is compoſed of two Triangles the Triangle ABC, is half the Parallelogram AGHC: Alſo the Triangle ACD is equal to half the Parallelogram ACIK, as was proved § V. Wherefore the Trapezium ABCD is equal to half the Parallelogram GHIK. To find the Area $HI = BF + DE$, therefore $\frac{1}{2} HI \times AC \, (= KI = GH) =$ Area of the Trapezium. Which was to be proved.

§ VII. *Of* IRREGULAR FIGURES.

IRREGULAR Figures are all ſuch as have more Sides than four, and the Sides and Angles unequal. All ſuch Figures may be divided into as many Triangles as there are Sides, wanting two. To find the Area of ſuch Figures, they muſt be divided into Trapeziums and Triangles, by Lines drawn from one Angle to another, and ſo find the Areas of the Trapeziums and Triangles ſeverally, and then add all the Areas together, ſo will you have the Area of the whole Figure,

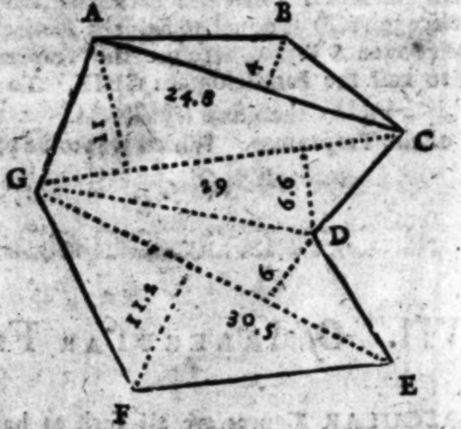
Let ABCDEFG be an irregular Figure given to be meaſured; firſt, draw the Lines AC and GD, and thereby divide the given Figure into two Trapeziums, ACGD and GDEF, and the Triangle ABC; of all which I find the Areas ſeverally.

Firſt, I multiply the Baſe AC by half the Perpendicular, and the Product is 49.6, the Area of the Triangle ABC.

Then, for the Trapezium, ACGD, the two Perpendiculars 11 and 6.6, added together make 17.6; the half thereof is 8.8, multiplied by 29, the Diagonal, the Product is 255.2, the Area of that Trapezium.

And for the Trapezium GDEF, the two Perpendiculars, 11.2 and 6, added together make 17.2, the half thereof is 8.6, which multiplied by 30.5, the Diagonal, the Product is 262.3, the Area thereof. All theſe Areas added together make 567.1, and ſo much is the Area of the whole irregular Figure.

See the Work.



24.8 Baſe AC.
2 half Perpendicular.
49.6 Area of ABC.

11 } Perpendiculars
6.6 }
17.6 Sum.
8.8 Half.
29 Diag. CG.
792
176
255.2 Area of ACGD.

11.2

11.2 } Perpendiculars.
6. }

17.2 Sum.
8.6 Half Sum,

30.5
8.6

1830
2440

262.30 Area of GDEF.
255.2 Area of ACGD.
49.6 Area of ABC.

567.1 Sum of the Areas.

The Figure being compoſed of Triangles and Trapeziums, and thoſe Figures being ſufficiently demonſtrated in the fifth and ſixth Sections aforegoing, it will be needleſs to mention any Thing of the Demonſtration thereof in this Place.

* * *

§ VIII. *Of* REGULAR POLYGONS.

Regular Polygons are all ſuch Figures as have more than four Sides, all the Sides and Angles thereof being equal Polygons, are denominated from the Number of their Sides and Angles.

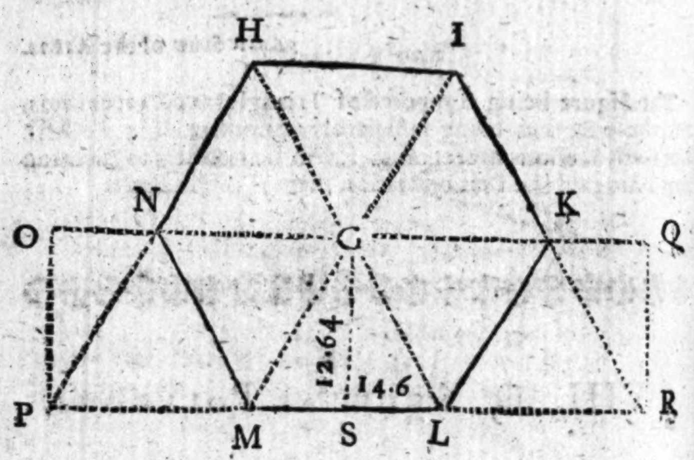
If the Figure conſiſts of 5, 6, 7, 8, 9, 10, 11, 12 Equal Sides and Angles, it is called a Regular Pentagon, Hexagon, Heptagon, Octagon, Enneagon, Decagon, Endecagon, Dodecagon.

5	Pentagon.
6	Hexagon.
7	Heptagon.
8	Octagon.
9	Enneagon.
10	Decagon.
11	Endecagon.
12	Dodecagon.

To find the Area or ſuperficial Content of any Regular Polygon, this is

The RULE.

Multiply the whole Perimeter, or Sum of the Sides, by half the Perpendicular, let fall from the Center to the Middle of one of the Sides; or multiply the half Perimeter by the whole Perpendicular, and the Product is the Area.



14.6
3

43.8 Half Sum of the Sides.

12.64 The Perpendicular.
43.8 Half Sum.

10112
3792
5056

553.632 Area.

14.6
6

87.6 Sum of the Sides.
6.32 Half Perpend.

1752
2628
5256

553.632 Area.

Let HIKLMN be a regular Hexagon, each Side thereof being 14.6, the Sum of all the Sides is 87.6, the half Sum thereof is 43.8, which multiplied by the Perpendicular GS 12.64, the Product is 553.63: Or if 87.6, the whole Sum of the Sides, be multiplied by half the Perpendicular 6.32, the Product is 553.632, the ſame as before; which is the Area of the given Hexagon.

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 12.6, that Extent will reach from 43.8, the ſame way to 553.63: Or extend from 2 to 12.6, that Extent will reach from 87.6 to 553.63.

DEMONSTRATION.

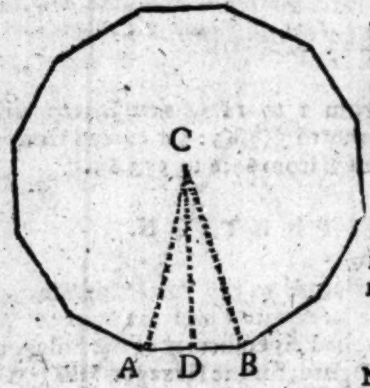
Every regular Polygon is equal to the Parallelogram, or long Square, whoſe Length is equal to half the Sum of the Sides, and Breadth equal to the Perpendicular of the Polygon, as appears by the foregoing Figure; for the Hexagon HIKLMN is made up of ſix equilateral Triangles: And the Parallelogram OPQR is alſo compoſed of ſix equal and equilateral Triangles, that is, five whole ones, and two half ones; therefore the Parallelogram is equal to the Hexagon.

A TABLE *for the more ready finding the Area of a Polygon.*

Number of Sides.	Names.	Multipliers.
3	Trigon	.433013
4	Tetragon	1.000000
5	Pentagon	1.720477
6	Hexagon	2.598076
7	Heptagon	3.633959
8	Octagon	4.828427
9	Enneagon	6.181827
10	Decagon	7.694209
11	Endecagon	8.514250
12	Dodecagon	9.330125

Multiply the Square of the Side by the tabular Number, and the Product is the Area of the Polygon.

How to find theſe tabular Numbers.



Theſe Numbers are found by Trigonometry, thus: Find the Angle at the Center of the Polygon, by dividing 360 Degrees by the Number of Sides of the Polygon.

Example. Suppoſe each Side of the Dodecagon annexed be 1, and the Area be required.

Divide 360 by 12, (the Number of Sides) the Quotient is 30 Degrees for the Angle ACB; the half thereof is 15, the Angle DCB, whoſe Complement to 90 Degrees is 75 Degrees, the Angle CBD: Then ſay,

As s. DCB 15 Degrees, ------ C. Ar. 0.587004
To .5, the half Side DB Log.-------- 1.698970
So is s. CBD 75 Degrees, ---------- 9.984944

To the Perpendicular CD 1.866025 0.270918

Then 1.866025 multiplied by .5, (the half Side) the Product is 9.330125, the Area of the Dodecagon required.

§ IX.

§ IX. *Of a* CIRCLE.

A Circle is a plain Figure, contained under one Line, which is called a Circumference, unto which all Lines, drawn from a Point in the Middle of the Figure, called the Center, and falling upon the Circumference thereof, are all equal the one to the other. The Circle contains more Space than any plain Figure of equal Compaſs.

Problem 1. Having the Diameter and Circumference, to find the Area.

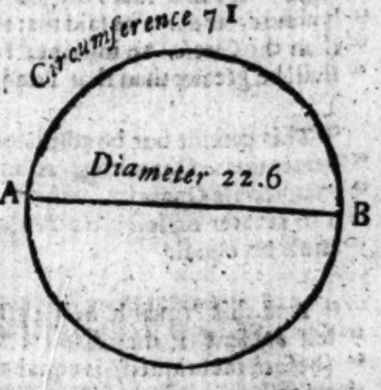
The RULE.

Every Circle is equal to a Parallelogram, whoſe Length is equal to half the Circumference, and the Breadth equal to half the Diameter; therefore multiply half the Circumference by half the Diameter, and the Product is the Area of the Circle.

35.5 Half Circumference
11.3 Half Diameter.

1065
355
355

401.15 Area.



Thus, if the Diameter of a Circle (that is, the Line drawn croſs the Circle thro' the Center) be 22.6, and if the Circumference be 71, the half of 71 is 35.5, and the half of 22.6 is 11.3; which multiplied together, the Product is 401.15, which is the Area of the Circle.

DEMONSTRATION.

Every Circle may be conceived to be a Polygon of an infinite Number of Sides, and the Semidiameter muſt be equal to the Perpendicular of ſuch a Polygon, and the Circumference of the Circle equal to the Periphery of the Polygon; therefore half the Circumference, multiplied by half the Diameter, gives the Area, as aforeſaid.

Or. (with *F. Ignat. Gaſton. Pardies*) "Every Circle is equal "to a Rectangle-Triangle, one of whoſe Legs is the Radius, "and the other a right Line equal to the Circumference of "the Circle: For ſuch a Triangle will be greater than any "Polygon inſcribed, and leſs than any Polygon circumſcribed, (by the 24th, 25th, 26th and 27th Articles of the "fourth Book of his Elements of Geometry) and therefore "muſt be equal to the Circle.

"For (ſays he) ſhould it be greater than the Circle, be the "Exceſs as little as it will, a Polygon may be circumſcribed, "whoſe Difference, from the Circle, ſhall be yet leſs than "the Difference between that Circle and the Rectangle Triangle; and that that Polygon will be leſs than the Triangle "is abſurd; and if it be ſaid that this rectangled Triangle is leſs "than the Circle, an inſcribed Polygon may be made which "ſhall be greater than that Triangle; which is impoſſible.

"This cannot but be admitted as a Principle, That if two "determinate Quantities, A and B, are ſuch, that if every "imaginable Quantity, which is greater or leſs than A, is "alſo greater or leſs than B, theſe two Quantities A and B "muſt be equal.

"And this Principle being granted, which is in a manner "ſelf evident, it may directly be proved, that the Triangle "(before mentioned) is equal to the Circle; becauſe every "imaginable inſcribed Figure, which is leſs than the Circle, "is alſo leſs than the Triangle; and every circumſcrib'd Figure "greater than the Circle, is alſo greater than the Triangle."

Problem 1. Having the Diameter of a Circle, to find the Circumference.

As 7 to 22, ſo is the Diameter to the Circumference.
Or, as 113 to 355, ſo is the Diameter to the Circumference.
Or, as 1 to 3.141593, ſo is the Diameter to the Circumference.

Let the Diameter (as in the former Circle) be 22.6, this multiplied by 22, and the Product is 497.2; which divided by 7, gives 71.028 for the Circumference. Or (by the ſecond Proportion) if 22.6 be multiplied by 355, the Product will be 8023; this divided by 113, the Quotient is 71, the Circumference. Or (by the third Proportion) if 22.6 be multiplied into 3.141593, the Product is 71.0000018, the Circumference; which two laſt Proportions are the moſt exact.

```
      22.6                 355
        22                22.6
     -----               -----
       452                2130
      452                 710
    ------               710
7)497.2(71.028           -----
                     113)8023.0(71
      3.141593            791
          22.6            ----
     ---------             113
      18849558             113
      6283186             ----
     6283186               ...
    ----------
     71.0000018
```

By Scale and Compaſſes.

Extend the Compaſſes from 7 to 22, or from 113 to 355, or from 1 to 3.14159; that Extent will reach from 22.6 to 71.

The Proportion of the Diameter of a Circle, to the Circumference, was never yet exactly found, notwithſtanding many eminent learned Men have laboured very far therein; amongſt which, the excellent *Van Culen* hath hitherto out done all, in his having calculated the ſaid Proportion to 36 Places of Decimals, which are engraven upon his Tomb-ſtone in St. *Peter's* Church in *Leyden*; which Numbers are theſe, Diameter.

1.00000.00000.00000.00000.00000.00000.00000

Circumference.

3.14159.26535.89793.23846.26433.83279.50288.

Of which large Number theſe ſix Places, 3.14159, anſwering to the Diameter 1.00000, may be ſufficient; of the three Proportions, as 7 to 22, 113 to 355, and 1 to 3.14159, I ſhall leave my Reader to uſe which of them he pleaſes, but ſhall commend the laſt two as moſt exact, tho' the firſt be moſt in Uſe; but in the following Work I ſhall uſe ſometimes one of them, and ſometimes another, but for the moſt part that of *Van Culen*, as being moſt exact.

Problem 3. Having the Circumference of a Circle to find the Diameter.

As 1 is to .318309, ſo is the Circumference to the Diameter.
Or, as 355 to 113, ſo is the Circumference to the Diameter.
Or, as 22 to 7, ſo is the Circumference to the Diameter.

Let the Circumference be 71, (as in the former Circle) if .318309 be multiplied by 71 (as by the firſt Proportion) the Product will be 22.599939 for the Diameter. Or, by the ſecond Proportion, 113 multiplied by 71, the Product is 8023; which divided by 355, the Quotient will be 22.6 the Diameter. Or, by the third Proportion, 71 multiplied by 7, the Product is 497; this divided by 22, the Quotient is 22.5909, the Diameter.

.318309	113	71
71	71	7
318309	113	22)497(22.59
2228163	791	44
22.599939	355)8023(22.6	57
	710	44
	923	130
	710	110
	2130	200
	2130	198
		2

Thus, by both the firſt Proportions, the Diameter is 22.6; but by the laſt it falls ſomething ſhort.

By Scale *and* Compaſſes.

Extend the Compaſſes from 3.14159 to 1, that Extent will reach from 71 to 22.6, which is the Diameter ſought.

Or you may extend from 1 to 318.309.

Or from 22 to 7.

Or from 355 to 113; the ſame will reach from 71 to 22.6 as before.

NOTE, That if the Circumference be 1, the Diameter will be .318309.

Problem 4. Having the Diameter of a Circle, to find the Area.

All Circles are in Proportion one to another, as are the Squares of their Diameters, (by *Euclid*. 12. 2.) Now the Area of a Circle, whoſe Diameter is 1, will be .785398, according to *Van Culen*'s Proportion, before mentioned; but for Practice .7854 will be ſufficient: Therefore,

As 1 (the Square of the Diameter 1) is to .7854, ſo is 510.76 (the Square of 22.6), the Diameter of the given Circle) to 401.51, (the Area of the given Circle) But, According to *Metius* Proportion;

As 452 : 355 :: 510.76 : 401.15, the ſame as before.

But, it you uſe *Archimedes*'s Proportion.

As 14 : 11 :: 510.76 : 401.31; which Area is greater than by the two former Proportions; tho' in ſmall Circles this is near enough the Truth.

See the Working of all theſe.

```
     22.6 Diameter of the former Circle.
     22.6
    ------
     1356
     452
    452
   -------
   510,76 The Square of the ſaid Diameter.

As 1 : .7854 :: 510.76
                 .7854
               -------
                204304
               255380
              408608
             357532
            -------
             401.150904 The Area.
```

By

By Scale *and* Compaſſes.

The Extent from 1 to 22.6, being twice turned over from .7854, will fall at the laſt upon 401.15, the Area.

113

4

452

As 452 : 355 :: 510.76

355

255380

255380

153228

452)181319.80(401.15

1808

519

452

678

452

2260

2260

....

As 14 : 11 :: 510.76

11

14)561836(401.31

56

18

14

43

42

16

14

2

Problem 5. Having the Circumference of a Circle, to find the Area.

Becauſe the Diameters of Circles are proportional to the Circumferences; that is, as the Diameter of one Circle is to its Circumference, ſo is the Diameter of another Circle to its Circumference. Therefore the Areas of Circles are to one another as the Squares of the Circumferences. And if the Circumference of a Circle be 1, the Area of that Circle will be .07958; then the Square of 1 is 1, and the Square of 71 (the Circumference of the former Circle) is 5041. Therefore it will be,

Sq Cir. Area Sq. Circumf.
As 1 : .07958 :: 5041
5041

7958
22 31832
4 397900

88 401.16278 Area.

Or thus:

As 88 : 7 :: 5041
7

88|35288|400.98 Area.
352

870
793

355 780
4 704

1424 76
Area.

Or, As 1420 : 113 :: 5041 : 401.15.

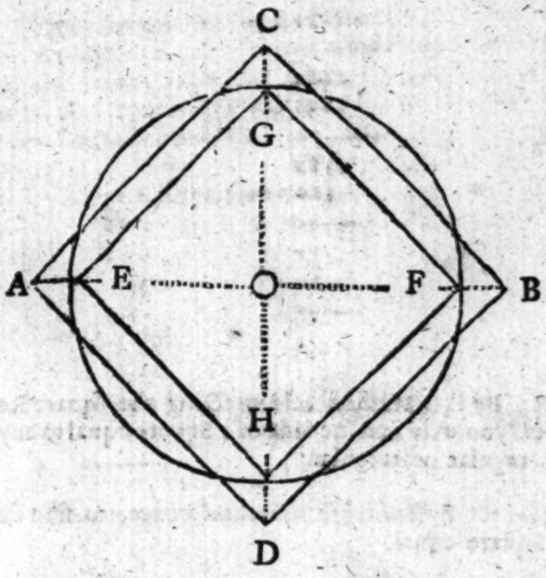
Problem 6. By having the Diameter, to find the Side of a Square that is equal in Area to that Circle.

If the Diameter of a Circle be 1, the Side of a Square equal thereunto will be .8862: Therefore,

As 1 : .8862 :: 22.6 (the Diameter.)
22.6

53172
17724
17724

To 20.02812 The Side of the Square AC.



Let the Diameter of the Circle EF or GH be 22.6 (as before) to find the Side of the Square AC, AD, *&c.* If .8862 be multiplied by 22.6, the Product is 20.02812, which is the Side of a Square, equal in Area to the Circle given; for if 20.02812 be multiplied ſquare-wiſe, that is, by itſelf, it will produce 401.1255907344, which is nearly equal to the Area found in the laſt *Problem.*

You may find the Side of the Square equal, by extracting the ſquare Root out of the Area of the given Circle.

401.15(20.0287295 Side of the Square.
4

4002)01.1500
8000

40048)349600
.... 320384

29216
28034

1182
801

381
360

21
20

1

N. B. By this Method of extracting the ſquare Root of the Area, you may find the Side of a Square equal to any plain Figure, regular or irregular.

Problem 7. By having the Circumference, to find the Side of the Square equal.

If the Circumference of a Circle be 1, the Side of the Square equal will be .2821. Therefore,

As 1 : .2821 :: 71 (the Circumference.)
71

2821
19747

20.0291 The Side of the Square.

Problem 8. Having the Diameter, to find the Side of a Square which may be inſcribed in that Circle.

If the Diameter of a Circle be 1, the Side of the Square inſcribed will be .7071. Therefore,

As 1 : .7071 :: 22.6
22.6

42426
14142
14142

To 15.98046 the Side EG inſcribed.

Or, if you ſquare the Semidiameter, and double that Square, the ſquare Root of the doubled Square will be the Side of the Square inſcribed. For (by *Euclid*, 1. 47.) the Square of the Hypothenuſe EG is equal to the Sum of the other two Legs, EO and OG.

11.3 Semidiameter.
11.3

339
113
113

127.69 The Squ. of EO, which double, becauſe
2 {EO=GO.

255.38(15.98 Root which is the Side of the Squ.
1

25(155
125

309)3038
2781

3188)25700
25504

196

***Problem* 9.** **Having the Circumference to find the Side of a Square which may be inſcribed.**

If the Circumference be 1, the Side of the Square inſcribed will be .2251. Therefore,

As 1 : 22.51 :: 71
71

2251
15757

15.9821 The Side of the Square EG.

Becauſe that in each of the four laſt *Problems. viz.* the 6th, 7th, 8th, and 9th, there is a Proportion laid down, it will be eaſy to work them with Scale and Compaſſes; for if you extend the Compaſſes from the firſt to the ſecond, that Extent will reach from the third to the fourth. As in the laſt *Problem*, where the Proportion is as 1 to .2251, ſo is 71 to the Side of the Square 15.9821. Here extend the Compaſſes from 1 to .2251; that Extent will reach from 71 to 15.98; and ſo of the reſt. But the 5th muſt be wrought like the fourth, thus; extend the Compaſſes from 1 to 71, that Extent turned over the ſame Way from .07958, will fall at the laſt upon 401.15.

Problem 10. Having the Area, to find the Diameter.

If the Area of a Circle be 1, the Square of the Diameter thereof is 1.2732. Therefore,

As

```
Area. Sq. Diam.  Area.           . . . . . .
As 1 : 1.2732 :: 401.15          510.744180(22.599 the Diam
       401.15                    4
      ------                     ----
       63660                   42)110
      12732                       84
     12732                       ----
    50928                     445)2674
   ---------                      2225
   510.744180                     -----
                              4509)44941
                                   40581
                                   -----
                              45189)436080
                                    406701
                                    ------
                                    .29379
```

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 1.2732; that Extent will reach from 401.15 to 510.74, *&c.* Then divide the Space between 1 and 510.74 into two equal Parts, and you'll find the middle Point at 22.6. Or you may divide the Space upon the Line of Numbers, between 401.55 and 78.54, into two equal Parts, and one of thoſe Parts will reach from 1 to 22.6, the Diameter ſought.

Problem 11. Having the Area, to find the Circumference.

If the Area of a Circle be 1, the Square of the Circumference will be 12.56637. Therefore,

```
Ar. Sq.Circum.  Area.           . . . . . . . Circumf.
As 1 : 12.56637 :: 401 15       5040.99932550(70.9999 Root
        401.15                  49
       -------                  ------
       6283185              1409)14099
     1256637                     12681
    1256637                      ------
   50265480                 14189)141893
  ------------                    127701
  5040.99932550                   ------
                            141989)1419225
                                   1277901
                                   -------
                            1419989)14132450
                                    12779901
                                    --------
                                    1352549
```

By Scale and Compaſſes.

Divide the Space between 401.15 and .07958, upon the Line, into two equal Parts; one of thoſe Parts will reach from 1 to 71, the Circumference ſought.

Problem 12. Having the Area, to find the Side of a Square inſcribed.

If the Area of a Circle be 1, the Area of a Square inſcribed within that Circle will be .6366. Therefore,

```
As 1 : 401.15 :: .6366
              .6366
        ----------
            240690
           240690
          120345
         240690
       -----------
       . . . . .
       255.372090(15.98 Root, which is the Side of the
       1                          [Square ſought.
      -----
     25)155
        125
       -------
    309)3037
        2781
       -------
   3188)25620
        25504
       -------
        11690
```

The ſame Reaſon may be given for the laſt Proportion that was given before for the Proportion of Circles to the Squares of their Diameters and Circumferences; for not only the Squares of the Diameters and Circumferences are in Proportion to the Circles they belong to, but alſo all Figures inſcribed or circumſcribed have the Squares of their like Sides proportional to the Circles they are inſcribed in, or circumſcribed about, and alſo to the Figures themſelves: The Square of any Side of one Figure is to the Area of that Figure, as the Square of the like Side of another ſimilar Figure is to the Area thereof, as you may find proved at large in *Euclid*, *Sturmius*'s *Matheſis Enucleata*, and other Authors, but will be too large to inſert in this Place.

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 401.15, that Extent will reach from .6366 to 255.37; the half Space between that and 1 is at 15.98, the Side of the Square.

Problem 13. Having the Side of a Square, to find the Diameter of the circumſcribing Circle.

If the Side of a Square be 1, the Diameter of a Circle that will circumſcribe that Square will be 1.4142. Therefore,

As 1 : 1.4142 :: 15.98
15.98

113136
127278
70710
14142

22.598916 The Diameter ſought.

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 1.4142, and that Extent will reach from 15.98 to 22.6, the Diameter ſought.

Problem 14. Having the Side of a Square, to find the Diameter of a Circle equal.

If the Side of a Square be 1, the Diameter of a Circle equal thereunto will be 1.128. Therefore,

Side. Diam. Side of a Square.
As 1 : 1.128 :: 20.0291
1.128

1602328
400582
200291
200291

22.5928248 Diameter.

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 1.128; that Extent will reach from 20.0291 (the Side of the Square given) to 22.6, the Diameter of the Circle ſought.

Problem 15. Having the Side of a Square, to find the Circumference of the circumſcribing Circle.

If the Side of a Square be 1, the Circumference of a Circle that will encompaſs the Square will be 4.443. Therefore,

Side Sq. Circum. Side Sq.
As 1 : 4.443 :: 15.98
15.98

35544
39987
22215
4443

70.99914 The Circum.

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 4.443, that Extent will reach from 15.98 to 71, the Circumference.

Problem 16. Having the Side of a Square, to find the Circumference of a Circle that will be equal thereunto.

If the Side of the Square be 1, the Circumference of a Circle that will be equal thereunto ſhall be 3.545. Then

As 1 : 3.545 :: 20.0291
3.545

1001455
801164
1001455
600873

71.0031595 The Circum.

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 3.545, that Extent will reach from 20.0291 to 71, the Circumference ſought.

In ſeveral of the foregoing *Problems*, where the Diameter and Circumference is requir'd, the Anſwers are not exactly the ſame as the Diameter and Circumference of the given Circle, but are ſometimes too little, as in the two laſt Problems, where the Anſwers in each ſhould be 71, the one being too much, and the other too little. The Reaſon of this is, the ſmall Defect that happens to be in the Decimal Fractions, they being ſometimes too great, and ſometimes too little; yet the Defect is ſo ſmall, that it is needleſs to calculate them to more Exactneſs.

* * *

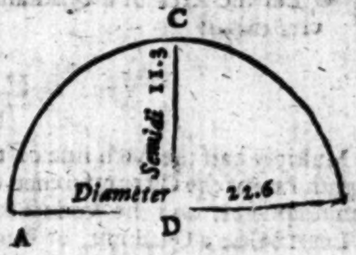
§ X. *Of a* SEMICIRCLE.

TO find the Area of a Semicircle, this is

The RULE.

Multiply the fourth Part of the Circumference of the whole Circle (that is, half the Arch-line) by the Semidiameter, the Product is the Area.

Let ABC be a Semicircle whoſe Diameter is 22.6, and the half Circumference, or Arch-line ACB, is 35.5, the half thereof is 17.75, which multiply by the Semidiameter 11.3, and the Product is 200.575, the Area of the Semicircle.



17.75 The half Arch-line.

11.3 The Semidiameter.

5325

1775

1775

200.575 The Area of the Semicircle.

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 11.3; that Extent will reach from 17.75 to 200.575, the Area.

If only the Diameter of the Semicircle be given, you may ſay, by the *Rule of Three,*

As 1 is to .3927, ſo is the Square of the Diameter to the Area.

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to the Diameter 22.6; that Extent turned twice from .3927, will reach at the laſt to 200.575.

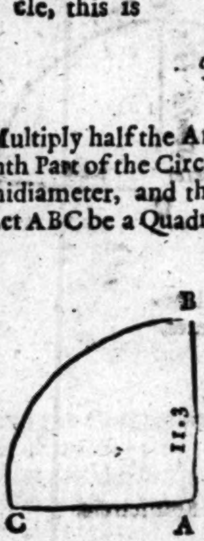
§. IX. *Of a* QUADRANT.

TO find the Area of a Quadrant, or fourth Part of a Circle, this is

The RULE.

Multiply half the Arch-line of the Quadrant, (that is, the eighth Part of the Circumference of the whole Circle) by the Semidiameter, and the Product is the Area of the Quadrant.

Let ABC be a Quadrant, or fourth Part of a Circle, whoſe Radius, or Semidiameter is 11.3, and the half Arch-line 8.875: theſe multiplied together, the Product is 100.2875 for the Area.

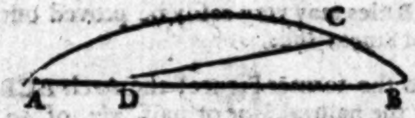


Theſe are the Rules and Ways commonly given for finding the Area of a Semicircle and Quadrant; but, I think, it is as good a Way to find the Area of the whole Circle, and then take half that Area for the Semicircle, and fourth Part for the Quadrant.

Before I proceed to ſhew how to find the Area of the Sector, and Segment of a Circle, I ſhall ſhew how to find the Length of the Arch-line both *Geometrically* and *Arithmetically.*

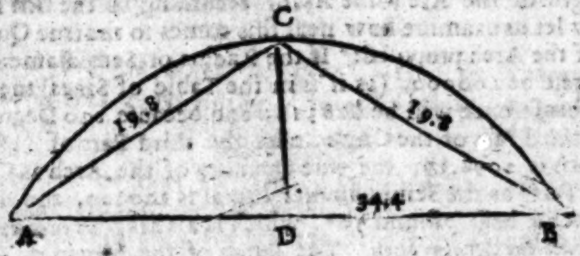
To find the Length of the Arch-line *Geometrically.*

Divide the Chord-line AB into four equal Parts, and ſet one of theſe Parts from B to C, and draw a Line from C to three of thoſe Parts at D; ſo ſhall CD be equal to half the Arch-line ACB.



To find the Length of the Arch line *Arithmetically.*

Multiply the Chord of half the Segment AC or CB by 8, and from the Product ſubtract the Chord of the whole Segment AB, and divide the Remainder by 3, the Quotient is the Arch-line ACB ſought.



19.8 AC
8
158.4
34.4 AB
3)124
Arch-line 41.333

Another

Another Way.

From the double Chord of half the Segment's Arch, ſubtract the Chord of the Segment, one third Part of the Difference added to the double Chord of half the Segment's Arch, the Sum is the Arch line of the whole Segment.

Thus, if AC 19.8 be doubled, it makes 39.6; from which if you ſubtract 34.4, the Remainder is 5.2, which divided by 3, the Quotient is 1.733; this added to 39.6 (the double Chord of the half Segment) the Sum is 41.333. So if the Arch-line ACB was ſtretched out ſtrait, it would then contain 41.333 ſuch Parts as the Chord AB contains 34.4 of the like Parts.

Theſe two Rules may very eaſily be proved out of the Table of natural Sines: thus,

Suppoſe (in the former Figure) the Arch ACB to contain 120 Degrees, the natural Sine of half, *viz.* of 60 Degrees, is 86602, which being doubled is 173204, which is the Chord of the whole 120 Deg. that is AB. Then, to find the Chord of the half Arch, *viz.* AC 60 Degrees, the half of it 30 Deg. the natural Sine thereof is 50000, which doubled makes 100000 for the Chord AC; then, according to the firſt Rule, multiply 100000 by 8, the Product is 800000; from which ſubtract 173204 (the Chord AB) and the Remainder is 626796, which divide by 3, the Quotient is 208932, which is the Length of the Arch-line ACB, according to the firſt Rule. Now let us examine how near this comes to the true Quantity of the Arch propoſed: If the Radius or Semidiameter of a Circle be 100000, (as it is in the Table of Sines) then the Circumference will be 628318; and becauſe 120 Degrees is the third Part of the Circle, take the third Part of 628318, which is 2094.39, the true Quantity of the Arch ACB in ſuch Parts as the Semidiameter contains 100000, and differs from that before found 507, which is a Thing inconſiderable in *Practical Menſuration.* The latter of the foregoing Rules agrees exactly with the former, and therefore the Difference will be the ſame as above; either of the Rules gives the Quantity of the Arch line too little; and the greater the Arch the greater the Error. If you know the Degrees that are contained in the Segment's Arch, and would have the Arch-line very exactly, you may reaſon thus by the *Rule of Three*;

As the Circle's Periphery in Degrees : is to its Periphery in equal Parts :: ſo is the Arch in Degrees and decimal Parts : to the ſame Arch in equal Parts.

Suppoſe the Circumference of a Circle be 71, and ſuppoſe the Arch to contain 52 Degrees, 15 Minutes, (the Decimal of 15 Minutes is .25) then ſay,

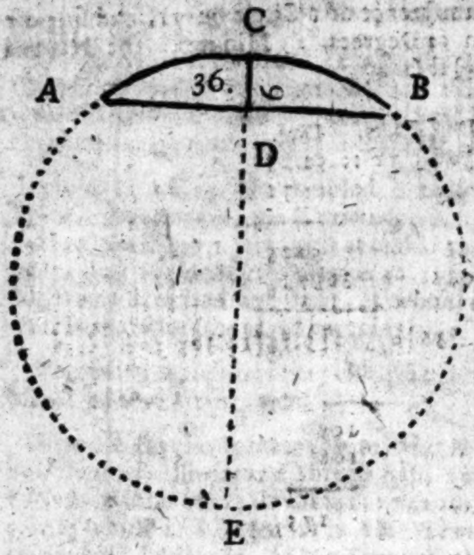
```
       Deg. Parts.  Deg.
     As 360 : 71 :: 52.25
                       71
                    -----
                     5225
                   36575
                  -------
        36|0)370|9.75(10.305 ferè.
             36
             --
              109
              108
              ---
                175
```

So the 52 Degrees, 15 Minutes, will contain 10.305 of ſuch Parts as the Circumference contains 71.

Thus have I ſhewed ſeveral Ways of finding the Meaſure of the Curve-line of any Part of a Circle very near the Truth. The next Thing I ſhall ſhew, is,

How to find the Diameter of a Circle, *by having the Chord and verſed Sine of the Segment, Arithmetically.*

Becauſe the Chord AB cuts the Diameter EC at right Angles, therefore the Semichord AD or DB is a mean proportional Line between the Parts of the Diameter CD and (DE, by *Euclid*, 6. 13.) Therefore, if you ſquare the Semichord AD or DB, and divide that Square by the verſed Sine CD, the Quotient will be the Part of the Diameter wanting; to which add the given verſed Sine CD, and the Sum is the Diameter ſought.



Example. Let ACB be a Segment given, whoſe Chord AB is 36, and the verſed Sine CD 6; half 36 is 18, which ſquared, makes 324; this divided by 6, the Quotient is 54; to which add 6, the Sum is 60, the Diameter of the Circle CE.

18 Half the Chord.
18

144
18

6)324 The Square of AD.

54 The Part wanting DE.
6 The verſed Sine CD add.

60 The Diameter CE.

§ XII. *Of a Sector of a* CIRCLE.

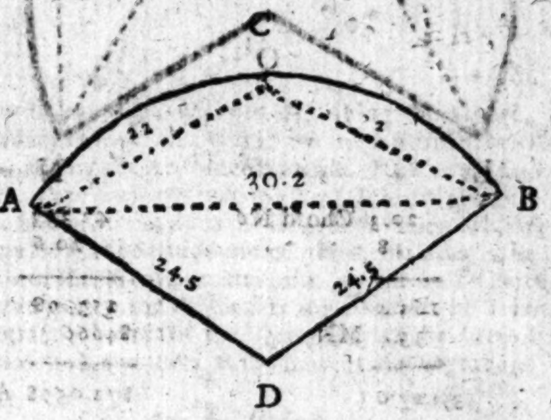
A Sector of a Circle is comprehended under two Radii, or Semidiameters, which are ſuppoſed not to make one right Line, and a Part of the Circumference; whence a Sector may be either leſs or greater than a Semicircle. To find the Area or ſuperficial Content thereof, this is

The

The RULE.

Multiply half the Arch-line by the Semidiameter, and the Product is the Area.

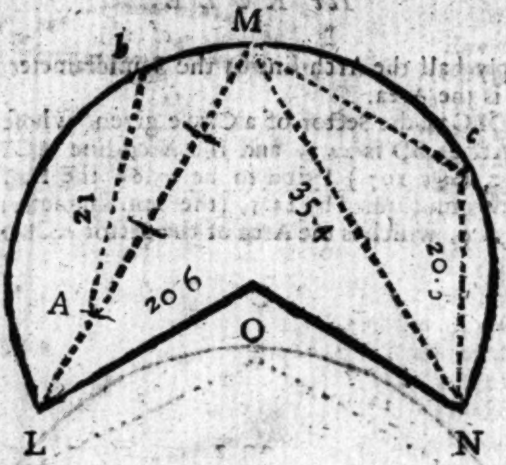
Let ADBC be the Sector of a Circle given, whoſe Semidiameter AD or BD is 24.5, and the Arch-line ACB (by the firſt Rule, *Page* 107.) I find to be 45.6, the Half thereof, 22.8, being multiplied by 24.5, (the Semidiameter) the Product is 558.6, which is the Area of the Sector ACBD.



22	22.8 Half Arch-line.
8	24.5 Semidiameter.
176	1140
39.2 Subtrahend.	912
	456
3)136.8	
	558.60 The Area.
45.6 Arch-line.	

Again. Let LMNO be a Sector greater than a Semicircle whoſe Semidiameter LO or NO is 20.6, and Line *b a* equal to a fourth Part of the Arch-line L *b* MN 21, the double whereof is 42, equal to the Arch-line L *b* M or M *c* N, or by the arithmetical Rule, *Page* 107, the ſaid Arch is found to be 42.333, which multiplied by 20.6, the Semidiameter makes 872.0598 for the Area of the Sector LMNO.

See the following Work.



20.3 Chord Nc
8
———
162.4
35.4 MN
———
3)127.0
———
42.333 Arch-line.

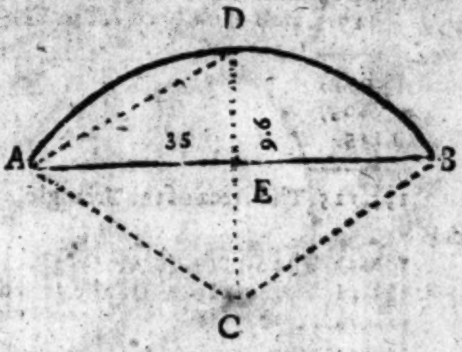
42.333
20.6
———
253998
84666
———
872.0598 Area.

* * *

§ XIII. *Of the Segment of a* CIRCLE.

A Segment of a Circle, is a Part terminated by a right Line leſs than the Diameter, called a Chord, and by a Part of the Circumference.

To find the Area of the Segment of a Circle, you muſt firſt find the Center of the whole Circle, and draw the two Semidiameters, thereby completing the Sector, as in the following Figure. Then (by the laſt Section) find the Area of the whole Sector CADBC, and then (by Sect. 5.) find the Area of the Triangle ABC, and ſubtract the Area of the Triangle out of the Area of the Sector, the Remainder is the Area of the Segment.



Otherwiſe you may, without deſcribing the Figure, find the Semidiameter of the Circle by the Arithmetical Rule, *Page* 110, and by the Arithmetical Rule, *Page* 107, find the Arch-line; then multiply half the Arch line by the Semidiameter, ſo have you the Area of the Sector. Then ſubtract the verſed Sine from the Semidiameter, the Remainder is the Perpendicular of the Triangle; and multiply the half Chord by the Perpendicular, the Product is the Area of the Triangle. Then ſubtract the Area of the Triangle from the Area of the Sector, and the Remainder is the Area of the Segment.

See the Work.

2)35=AB.

17.5
17.5

875
1225
175

96)306.15 (31.9
9.6 add
182
865 41.5 The Diameter of the Circle.

1 20.75 The Semidiameter.
9.6 DE Subtrahend.

11.15 Remains the Perpendicular EC.

11.15 The Perpendicular EC.
17.5 Half the Chord AE or EB.

5575
7805
1115

195.125 The Area of the Triangle.

306.25 The Square of AE.
92.16 The Square of DE the verſed Sine.

398.41 Sum.

The Square Root thereof is 19.96 the Chord AD.
8

159.68
Subtr. 35 The Chord AB.

3)124.68

2(41.56 The Arch-line

20.78 Half.
20.75 Semidiameter.

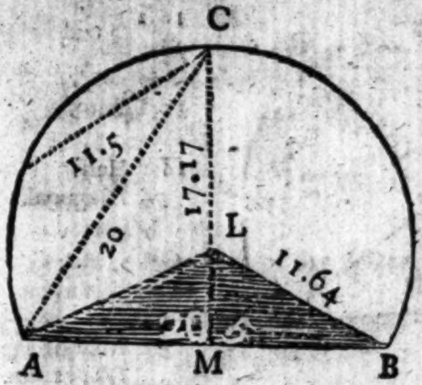
10390
14546
4156

From 431.1850 Area of the Sector.

Subtract 195.125 Area of the Triangle.

Remains 236.060 Area of the Segment.

Again. Let MACBM be a Segment greater than a Semicircle; obſerve the former Rules in all Reſpects, as in the laſt *Example*, only inſtead of ſubtracting the Area of the Triangle out of the Area of the Sector, here you muſt add it thereunto, as may plainly appear by the following Figure.



11.5
8

92.0
20

3)72

24 Half Arch-line

11.64 Semidiam.
24

4656
2328

17.17
11.64

5.53 Area of the Sector LACBL. 279.36 LM.
10.25 Half the Baſe MA.
5.53 The Perpendicular LM.

3075
5125
5125

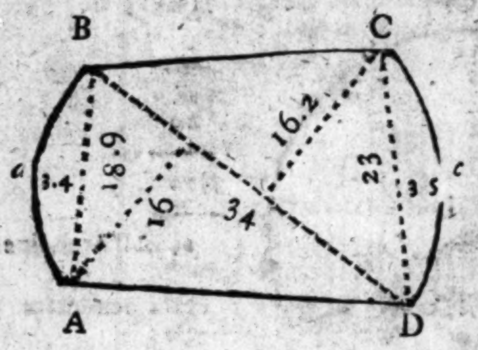
56.6825 The Area of the Triangle ALM.
279.36 The Area of the Sector add.

336.0425 The Area of the Segment ſought.

§ XIV. *Of* COMPOUND FIGURES.

MIXED or compound Figures, are ſuch as are compoſed of rectilineal and curvilineal Figures together.

To find the Area of ſuch mixed Figures, you muſt find the Area of the ſeveral Figures of which the whole compound Figure is compoſed, and add all the Areas together, and the Sum will be the Area of the whole compound Figure.



16.2 } add.
16.

32.2 Sum.

16.1 Half.
34 Diagonal.

644
483

547.4 Area of the (Trapezium.

10.236 Half the Arch-line A*a*B.
14.83 Semidiameter of the Arch A*a*B.

30708
81888
40944
10236

151.79988 Area of the Sector.

From 14.83 Semidiameter.
Subtract 3.4 Verſed Sine.

Rem. 11.43 Perpen. of the Triangle.
9.45 Half the Chord AB.

5715
4572
10287

108.0135 The Area of the Triangle ſubtracted.
151.7999 From the Area of the Sector.

43.7864 The Area of the Segment A*a*BA.

12.19 Half the Arch-line CeD.
20.64 Semidiameter.

4876
7314
2438

251.6016 The Area of the Sector.

From 20.64 The Semidiameter.
Subtract 3.5 Verſed Sine.

Remaind. 17.14 Perpendic. of the Triangle.
11.5 Half the Chord DC.

8570
1714
1714

Subtr. 197.110 Area of the Triangle.
From 251.602 The Area of the Sector.

Rem. 54.492 The Area of the Segment CeDC.
43.786 The Area of the Segment AaBA.
547.4 The Area of the Trapezium.

Sum 645.678 The Area of the Whole.

* * *

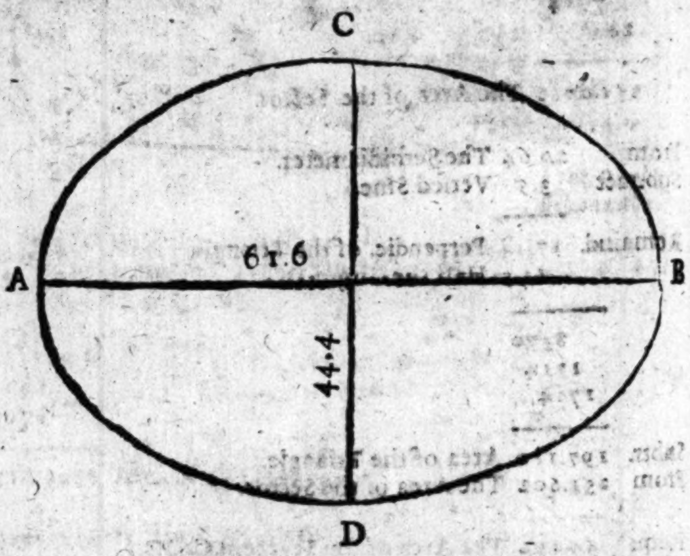
§ XV. *Of an* ELLIPSIS.

AN ELLIPSIS, or OVAL, is a Figure bounded by a regular Curve Line, returning into itſelf; but of its two Diameters, cutting each other in the Center, one is longer than the other, in which it differs from the Circle. To find the Area thereof, this is

The

The RULE.

Multiply the tranſverſe Diameter by the Conjugate, and multiply that Product by .7854, this laſt Product is the Area of the Oval.



61.6 The tranſverſe Diam.
44.4 The conjugate Diam.

2464
2464
2464

2735.04 The Rectangle.
.7854 The Area of Unity.

1094016
1367520
2188032
1914528

2148.100416 The Area of the Oval.

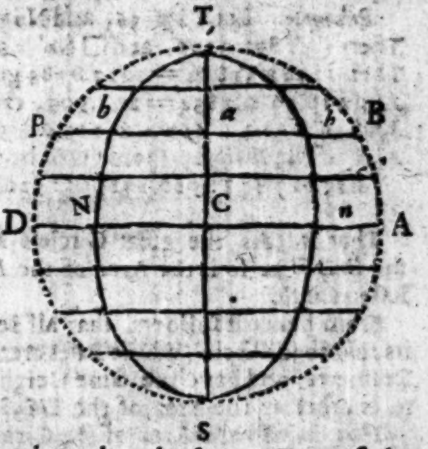
Demonſtration. If you circumſcribe any *Ellipſis* with a Circle, and ſuppoſe an infinite Number of Chord lines drawn therein, all parallel to the conjugate Diameter, as thoſe in the following Figure, then it will be,

As DA the Diameter of the Circle is : to Nn, the conjugate Diameter of the *Ellipſis* : : ſo is BaB, any Chord in the Circle, : to *bab*, its reſpective Ordinate in the *Ellipſis*.

For, according to the Property of the Circle,

it is | 1 | $aS \times Ta = \square Ba$.
And | | by the Property of the Ellipſis.
it is | 2 | $\square TC : NC :: aS \times Ta : \square ba$.
1, 2, | 3 | $\square TC : \square NC :: \square Ba : \square ba$.
3, hence | 4 | $TC . NC :: Ba : ba$.
Conſeq. | 5 | $2TC : 2NC :: 2Ba : 2ba$.
That is, | 6 | $DA : Nn :: BaB : bab$.

But the Sum of an infinite Series of ſuch Chords as Bab do conſtitute the Area of the Circle. And the Sum of the like Series of their reſpective Ordinates, as *bab*, do conſtitute the Area of the *Ellipſis.*



Therefore, As TS, to N*n* : : Circle's Area : to the *Ellipſis* Area. But TS : N*n* :: □TS : TS x N*n*; whence it follows, that,

As □TS : Circle's Area :: ſo is TS x N*n* : *Ellipſis* Area.

Conſequently, As 1 to .7854 :: ſo is the Rectangle, or Product of the tranſverſe and conjugate Diameter of any *Ellipſis* to its Area.

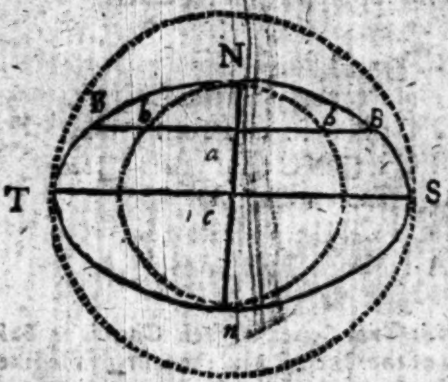
Hence it is eaſy to conceive, that the ſquare Root of the Product of the tranſverſe and conjugate Diameters, will be the Diameter of a Circle equal to the *Ellipſis.*

Hence alſo all Segments of an *Ellipſis*, and its circumſcribing Circle (whoſe Baſes are parallel to the conjugate Diameter, and of the ſame Height) are in Proportion one to another as their Baſes are; that is,

As B*a*B : *bab* : : Area Segment BTB : Area Segment *b*T*b*.

Or, As TS : N*n* : : Area Segment BTB : Area Segment *b*T*b*.

The Area of every *Ellipſis* is a mean Proportional between the Areas of its circumſcribing and inſcribed Circles.



The Truth of this may be eaſily deduced from the laſt; for it's already proved, that □ TS : TS x Nn :: circumſcribing Circle's Area : *Ellipſis* Area.

But □ TS : TS x Nn :: TS x Nn : □ Nn. Therefore *Ellipſis* Area : inſcribed Circle's Area :: TS x Nn : □ Nn.

Example. Let TS = 36, and Nn = 18.4
Then □ TS = 1296, and □ Nn = 338.56.
Then 1296 x .7854 = 1017.8784 great Circle's Area;
And 338.56 x .7854 = 265.905, *&c.* leſſer Circle's Area;
And 36 x 18.4 = 662.4 x .7854 = 520.24896, which is the Area of the *Ellipſis*; then it will be,

As 1017.878 : 520.24896 :: 520.24896 : 265.905084.

That is, As the great Circle's Area : is to the Area of the *Ellipſis* :: ſo is the Area of the *Ellipſis* to the Area of the leſſer Circle.

From hence it follows, that all Segments of an *Ellipſis* and its inſcribed Circle (whoſe Baſes are parallel to the tranſverſe Diameter, and have the ſame Height) are in Proportion one to another as the Area of the *Ellipſis* and Circle are.

That is, As the Area of the Circle : is to the Area of the *Ellipſis* :: ſo is the Segment *bNb* : to the Segment BNB.

Or, Nn : TS :: Area Segment *bNb* : Area Segment BNB.

* * *

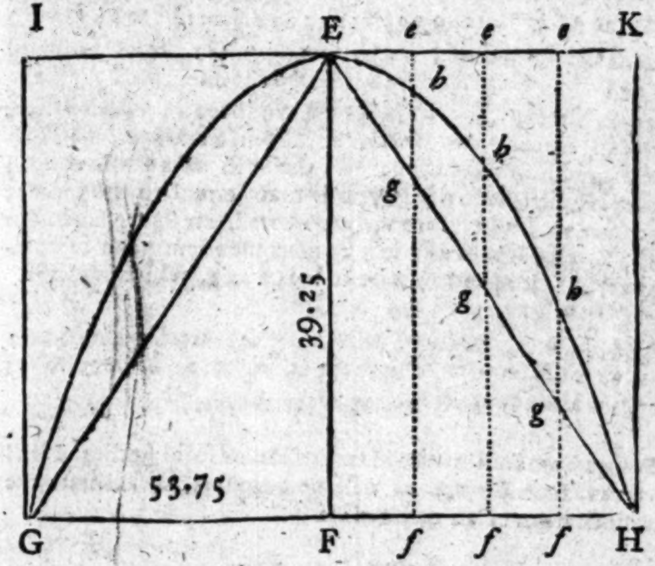
§ XVI. *Of a* PARABOLA.

A PARABOLA is a curvilinear Figure, made by the Section of a Cone, being cut by a Plane parallel to one of its Sides.

Every Parabola is two Thirds of its circumſcribing Parallelogram; therefore to find the Area thereof, this is

The RULE.

Multiply the Baſe, or greateſt Ordinate, by the perpendicular Height, and multiply that Product by 2, and divide the laſt Product by 3, the Quotient will be the Area of the Parabola.



```
    53.75 The Ordinate GH.
    39.25 The Perpendic. EF.
   ------
    26875
   10750
  48375
 16125
 ----------
 2109.6875
         2
 ----------
3)4219.3750
 ----------
 1406.4583 The Area.
```

Demonſtration. Let FH, the Semi-ordinate, be divided into four equal Parts, or into 8.16, *&c.* and thro' the Diviſions draw Lines, as *ef*, *ef*, *&c.* parallel to the Axis EF: Suppoſe alſo EF to be 4.

Then I ſay, the parabolick Space E*b*HF is to the Parallelogram EKFH as 2 to 3; but to the Triangle EFH as 4 to 3.

M For,

For, firſt, *gf*, *gf*, *gf*, &c. are in continual arithmetical Proportion from the Nature of plain Triangles.

Secondly, *fe* : *ge* :: *ge* : *he*; but *he* in the Axis EF = 0, and nthe firſt Parallel *ef* it muſt be equal to $\frac{1}{4}$ in the next *ef* it muſt be equal to $\frac{4}{4}$ in the third to $\frac{9}{4}$, and ſo on, in a duplicate arithmetical Progreſſion.

For as *ef* (= 4) : to *ge* (= 1) :: *ge* (= 1) : to *eh* (= $\frac{1}{4}$) And as the ſecond *ef* (= 4) : to *eg* (= 2) :: ſo is *eg* (= 2) : to *eh* (= 1) &c. And thus it will be if the Lines F*f*, *ff*, &c be again biſected, &c. *ad infinitum*. So that all the Indiviſibles of the trilinilear Space EKH*h*E will be in a duplicate arithmetical Progreſſion increaſing: But the Sum of a Rank of ſuch Terms is ſubtriple to a Rank of as many equal to the greateſt (by *Lemma* 3) wherefore the whole trilinear Space EKH*h*E is to the Parallelogram as 1 to 3; and conſequently the remaining parabolick Space muſt be to it as 2 to 3, which was, &c.

And ſince the Triangle FEH is to the Parallelogram as 1 to 2, it muſt be to the Parabola as 1$\frac{1}{2}$ to 2, or as 3 to 4; which was to be proved.

Before I proceed to the Menſuration of ſolid Bodies, I will lay down ſuch *Lemmas* as will be neceſſary to facilitate the Demonſtration of all ſuch Solids.

LEMMA I.

In any Series of equal Numbers, (repreſenting Lines or other Quantities) as 1, 1, 1, 1, &c. or 2, 2, 2, 2, &c. or 3, 3, 3, 3, &c. if one of the Terms be multiplied into the Number of Terms, the Product will be the Sum of all the Terms in the Series.

LEMMA II.

If a Series of Numbers, in arithmetical Progreſſion, begin with a Cypher, and the common Difference be 1, as 0, 1, 2, 3, &c. (repreſenting a Series of Lines or Roots beginning with a Point) if the laſt Term be multiplied into the Number of Terms, the Product will be double the Sum of all the Series.

That is, putting L = the laſt Term, N = the Number of Terms, and S = the Sum of all the Series. Then will NL = 2S; conſequently, ½ NL = S, *viz.* one half of ſo many times the greateſt Term as there are Numbers of Terms in the Series:

$$\text{Thus, } \begin{cases} 0+1+2+3+4 \\ \hline 4+4+4+4+4 \end{cases} = \frac{10 \text{ the Sum} = \frac{1}{2} \text{NL.}}{20 = \text{NL.}}$$

LEMMA III.

If a Series of Squares, whoſe Sides or Roots are in arithmetical Progreſſion, beginning with a Cypher, *&c.* be infinitely continued, the laſt Term being multiplied into the Number of Terms, will be triple to the Sum of all the Series, *viz.* NLL = 3S; or $\frac{1}{3}$ NLL = S.

That is, the Sum of ſuch a Series will be one Third of the laſt, or greateſt Term, ſo many times repeated as there are Numbers of Terms in their Series.

Inſtances in ſquare Numbers.

$$1 \begin{cases} \dfrac{0+1+4}{4+4+4} \end{cases} = \frac{5}{12} = \frac{1}{3} + \frac{1}{12}.$$

$$2 \begin{cases} \dfrac{0+1+4+9}{9+9+9+9} \end{cases} = \frac{14}{36} = \frac{7}{18} = \frac{1}{3} + \frac{1}{18}.$$

$$3 \begin{cases} \dfrac{0+1+4+9+16}{16+16+16+16+16} \end{cases} = \frac{30}{80} = \frac{3}{8} = \frac{9}{24} = \frac{1}{3} + \frac{1}{24}.$$

From theſe Inſtances it is evident, that as the Number of Terms in the Series do increaſe, the Fraction or Exceſs above one Third does decreaſe, the ſaid Exceſs always being $\frac{1}{6N-6}$, which, if we ſuppoſe the Series to be infinitely continued, will quite vaniſh, and become nothing at all.

LEMMA IV.

If a Series of Cubes, whoſe Roots are in arithmetical Progreſſion, beginning with a Cypher, *&c.* (as above) be infinitely continued, the Sum of all the Series will be one Fourth NLLL = S.

That is, one Fourth of the laſt Term ſo many times repeated as there are Numbers of Terms.

Inſtances in Cube Numbers.

If 0, 1, 2, 3, 4, 5, *&c.* be the Roots of the Cubes,

$$1\left\{\frac{0+1+8+27}{27+27+27+27}=\frac{36}{108}=\frac{4}{12}=\frac{1}{4}+\frac{1}{12}.\right.$$

$$2\left\{\frac{0+1+8+27+64}{64+64+64+64+64}=\frac{100}{320}=\frac{10}{32}=\frac{5}{16}=\frac{1}{4}+\frac{1}{16}\right.$$

$$3\left\{\frac{0+1+8+27+64+125}{125+125+125+125+125+125}=\frac{225}{750}=\frac{45}{150}=\frac{6}{20}=\frac{1}{4}+\frac{1}{20}.\right.$$

From theſe *Examples* it plainly appears, that as the Number of Terms in the Series increaſes, the Fraction or Exceſs above one Fourth decreaſes, the Exceſs being always $\frac{1}{4N-4}$ which, if we ſuppoſe the Series to be infinitely continued, will become infinitely ſmall, or nothing.

LEMMA V.

If a Series of Biquadrats, whoſe Roots are, in arithmetical Progreſſion, beginning with a Cypher, *&c.* as before, be infinitely continued, the Sum of all the Terms in ſuch a Series will be one Fifth NLLLL.

The Truth of this may be manifeſt by the like Proceſs, as in the foregoing *Lemmas*, and ſo on for higher Powers.

LEMMA

LEMMA VI.

The Sum of an infinite Progreſſion, whoſe greateſt Term is a ſquare Number, the others decreaſing by odd Numbers, *viz.* 1, 3, 5, *&c.* is in ſubſeſquialtern Proportion of the Sum of the like Number of equal Terms, that is, as 2 to 3.

Inſtances in ſuch Progreſſions.

$$1\left\{\frac{9+8+5}{9+9+9}\right.=\frac{22}{27}=\frac{2}{3}+\frac{4}{9}.$$

$$2\left\{\frac{16+15+12+7}{16+16+16+16}\right.=\frac{50}{64}=\frac{2}{3}+\frac{11}{32}.$$

$$3\left\{\frac{25+24+21+16+9}{25+25+25+25+25}\right.=\frac{95}{125}=\frac{2}{3}+\frac{7}{27}.$$

$$4\left\{\frac{36+35+32+27+20+11}{36+36+36+36+36+36}\right.=\frac{161}{216}=\frac{2}{3}+\frac{17}{72}.$$

From theſe *Examples* it plainly appears, that as the Number of Terms in the Series increaſes, the Fraction or Exceſs above $\frac{2}{3}$ decreaſes; and if we ſupppoſe the Series to be infinitely continued, that Exceſs will quite vaniſh, and the Sum of the infinite Series will be $\frac{2}{3}$ of ſo many equal to the greateſt.

CHAP. II.

The Menſuration *of* SOLIDS.

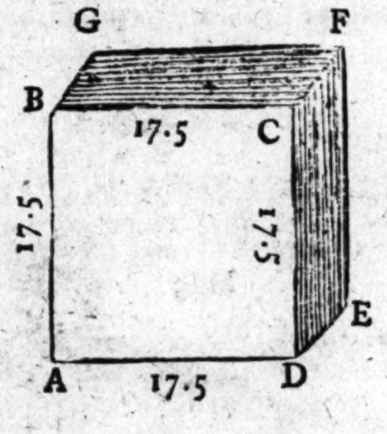
SOLID Bodies are ſuch as do conſiſt of Length, Breadth, and Thickneſs; as Stone, Timber, Globes, Bullets, *&c.*

§ I. *Of a* CUBE.

A CUBE is a ſquare Solid, comprehended under ſix geometrical Squares, being in the Form of a Dye. To find the ſolid Content, this is

The RULE.

Multiply the Side of the Cube into itſelf, and that Product again by the Side, the laſt Product will be the Solidity, or ſolid Content of the Cube.



17.5

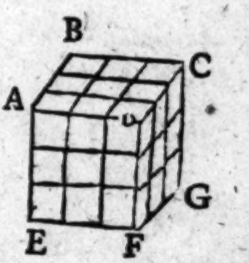
```
      17.5
      17.5
    ------
       875
     1225
     175
   -------
    306.25
      17.5
  --------
   153125
  214375
  30625
  --------
  5359.375 Solid Content of the Cube
```

Suppoſe ABCDEFG a cubical Piece of Stone or Wood, each Side thereof being 17 Inches and an half; multiply 17.5 by 17.5, and the Product is 306.25; which being multiplied by 17.5, the laſt Product is 5359.375, which is 5359 ſolid Inches, and 375 Parts. To reduce the ſolid Inches to Feet, divide by 1728 (becauſe ſo many cubical Inches is a Foot) and the ſolid Feet in the Cube will be 3, and 175 cubical Inches remain.

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 17.5, that Extent turned over twice from 17.5, will reach to 5359, the ſolid Content in Inches. Then extend the Compaſſes from 1728 to 1, that Extent turned the ſame Way from 5359, will reach to 3.1 Feet.

DEMONSTRATION.



If the Square ABCD be conceived to be moved down the Plain ADEF, always remaining parallel to itſelf, there will be generated, by ſuch a Motion, a Solid having ſix Plains, the two oppoſite whereof will be equal and parallel to each other; whence it is call'd a Parallelopipedon, or ſquare Priſm. And if the Plain ADEF be a Square equal to the generating Plain

ABCD, then will the generated Solid be a Cube. From hence ſuch Solids may be conceived to be conſtituted of an infinite Series of equal Squares, each equal to the Square ABCD; and AE or DF will be the Number of Terms. Therefore, if the Area of ABCD be multiplied into the Number of Terms, AE the Product is the Sum of all the Series, (*per Lemma* I.) and conſequently the Solidity of the Parallelopipedon or Cube. Or, if the Baſe ABCD, being divided into little ſquare Areas, be multiplied into the Height AE, divided by a like Meaſure for Length, after this Way you may conceive as many little Cubes to be generated in the whole Solid, as is the Number of the little Areas of the Baſe multiplied by the Number of Diviſions the Side AE contains. Thus, if the Side of the Baſe AB be 3, that multiplied into itſelf is 9, which is the Area of the ſquare Baſe ABCD; then, if AE be likewiſe 3, multiply 9 by 3, and the Product is 27; and ſo many little Cubes will this Solid be cut into, if you conceive it to be cut as the Lines direct.

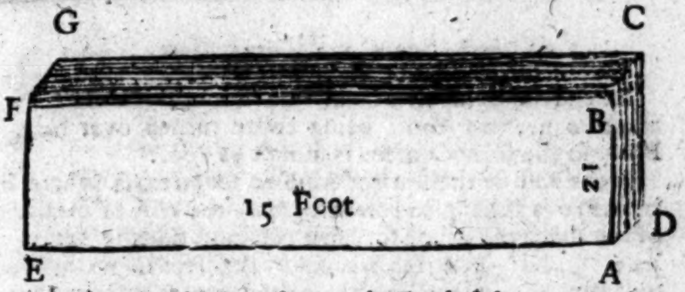
From this Demonſtration it is very plain, that if you multiply the Area of the Baſe of any Parallelopipedon into its Length or Height, that Product will be the ſolid Content of ſuch a Solid.

* * *

§ II. *Of a* PARALLELOPIPEDON.

LET ABCDEFG be a Parallelopipedon, or ſquare Priſm, repreſenting a ſquare Piece of Timber or Stone, each Side of its ſquare Baſe ABCD being 21 Inches, and its Length AE 15 Foot.

First,



Firſt, then, multiply 21 by 21, the Product is 441, the Area of the Baſe in Inches; which multiplied by 180, the Length in Inches, and the Product is 79380, the ſolid Content in Inches. Divide the laſt Product by 1728, and the Quotient is 45.9, that is, 45 ſolid Feet, and 9 Tenths of a Foot. Or thus, Multiply 441 by 15 Feet, and the Product is 6615; divide this by 144, and the Quotient is 45.9, the ſame as before.

Or thus, by multiplying Feet and Inches.

Multiply 1 Foot 9 Inches by 1 Foot 9 Inches, and the Product is 3 Feet, 0 Inches, 9 Parts; this multiplied again by 15 Feet, gives 45 Feet, 11 Inches, 3 Parts, that is, 45 Feet, and $\frac{11}{12}$ of a Foot, and $\frac{1}{4}$ of $\frac{1}{12}$.

	See the Work of all theſe,	F.	I.	
21	441	1	9	
21	15	1	9	
21	2205	1	9	
42	441	1	3	9
441	144)6615(45.9	3	0	9
180		15		
	855			
35280	1350	45	0	0
441			7	6
	54		3	9
1728)79380(45.9				
6912		45	11	3
10260				
8640				
16200				
15552				
648				

By Scale *and* Compaſſes.

Extend the Compaſſes from 12 to 21, and that Extent will reach to near 46 Foot, being twice turned over from 15 Foot; ſo the ſolid Content is almoſt 46 Foot.

If the Baſe of the ſquared Solid be not an exact Square, but in form of a rectangled Parallelogram, the Way of meaſuring of it is much the ſame; for, firſt you muſt find the Area of the Baſe, by multiplying the Breadth by the Depth, and then multiply that Area by the Length of the Piece, as before; thus,

If a Piece of Timber be 25 Inches broad, 9 Inches deep, and 25 Feet long, how many ſolid Feet are contained therein?

25
9
225
25
1125
450
144)5625(39
432
1305
1296
09

F. I.
2 1
0 9
1 6 9
25
25 0 0
12 6 0
1 0 6
0 6 3
39 0 9

Anſwer. 39 Feet.

By Scale *and* Compaſſes.

Firſt, find a mean geometrical Proportional between the Breadth and the Depth; which to do upon the Line of Numbers, you muſt divide the Space upon the Line, between the Breadth and Depth, into two equal Parts; that middle Point will be the mean Proportional ſought; thus the middle Point between 25 and 9 is at 15; ſo is 15 a mean Proportional between 9 and 25; for, as 9 : 15 :: 15 : 25; ſo a Piece of Timber of 15 Inches ſquare, is equal to a Piece 25 Inches broad and 9 Inches deep. So then, if you extend the Compaſſes from 12 to 15, that Extent turned twice over from 25 Feet, the Length, will reach to 39 Feet, the Content.

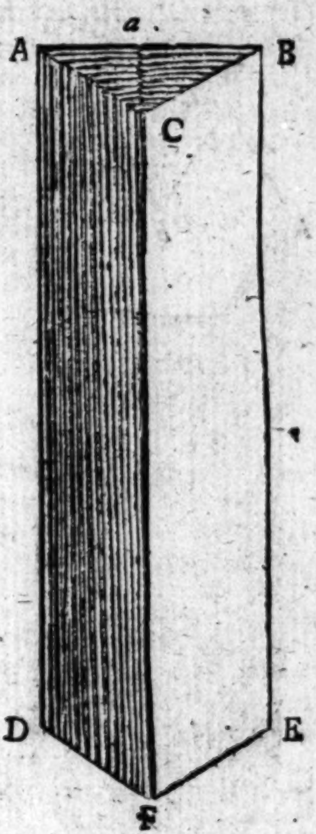
§ III.

§ III. *Of a Triangular* PRISM.

A Priſm is a Solid contained under ſeveral Plains, and having its Baſes alike, equal, and parallel: The ſolid Content of a Priſm (whether triangular or multangular) is found by multiplying the Area of the Baſe into the Length or Height, and the Product is the ſolid Content.

Let ABCDEF be a triangular Priſm, each Side of the Baſe being 15,6 Inches, the Perpendicular thereof C*a* is 13.51 Inches, and the Lenth of the Solid 19.5 Feet.

Multiply the Perpendicular of the Triangle 13.51, by half the Side 7.8, and the Product is 105.378, the Area of the Baſe; which multiply by the Length 19.5, and the Product is 2054.871, which divide by 144, and the Quotient is 14.27 Feet, *ferè*, the ſolid Content.



13.51
7.8

10808
9457

105.378
19.5

526890
948402
105378

2054.8710

144)2054.87(14.27
144

614
576

388
288

1007
1008

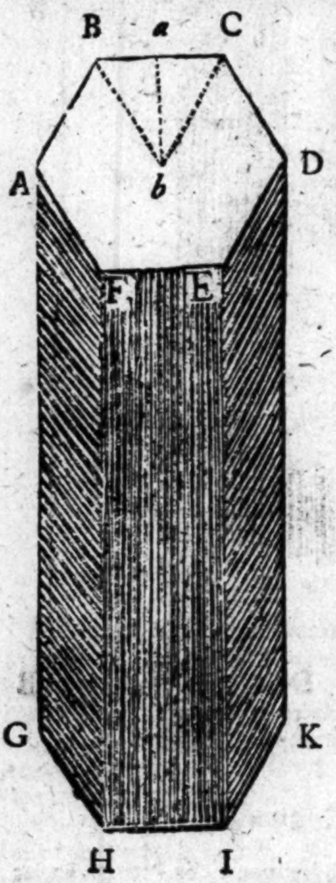
By Scale *and* Compaſſes.

Firſt, find a mean Proportional between the Perpendicular and half Side, (as before taught) by dividing the Space upon the

Line, between 13.51 and 7.8, into two equal Parts; ſo ſhall you find the middle Point between them to be at 10.26, which is the mean Proportional ſought: By this Means the triangular Solid is brought to a ſquare one, each Side being 10.26 Inches.

Then extend the Compaſſes from 12 to 10.26, that Extent turn'd twice downwards from 19.5 Feet, the Length, will at laſt fall upon 14.27, which is 14 Feet and a little above a Quarter.

Let ABCDEFGHIK repreſent a Priſm whoſe Baſe is a Hexagon, each Side thereof being 16 Inches, and the Perpendicular, from the Center of the Baſe to the Middle of one of the Sides (*ab*) is 13.84 Inches, and the Length of the Priſm is 15 Feet; the ſolid Content is required.



Multiply half the Sum of the Sides 48 by 13.84, and the Product is 664.32, the Area of the hexagonal Baſe, (by § 8. *p.* 86.) which multiply by 15 Feet, the Length, the Product is 9964.8, which divided by 144, the Quotient will be 69.2 Feet, the ſolid Content required.

13.84
48

11072
5536

664.32 Area of Baſe.
15

332160
66432

144)9964.80(69.2
864

1324
1296

288
288

..

By Scale *and* Compaſſes.

Firſt, find a mean Proportional between the Perpendicular and half the Sum of the Sides, that is, divide the Space between 13.84 and 48, and the middle Point will be 25.77; then extend the Compaſſes from 12 to 25.77, that Extent will reach (being twice turned over) from 15 Feet, the Length, to *69.2* Feet, the Content.

To find the ſuperficial Content of any of the forementioned Solids, you muſt take the Girt of the Piece, and multiply by the Length, and to that Product add the two Areas of the Baſes, the Sum will be the whole ſuperficial Content. *Example* of the Hexagonal Priſm laſt mention'd: The Sum of the Sides being *96*, and the Length 15 Feet, that is, 180 Inches, which multiplied by *96*, the Product is 17280 ſquare Inches, to which add twice *664.32*, the Areas of the two Baſes, and the Sum is 18608.64, the Area of the Whole, which is 129.32 Feet.

180
96
1080
1620
17280
664.32
664.32

144)18608.64(129.22
420
1328
326
384
96

The ſuperficial Content of the whole Solid is 129.22 Feet.

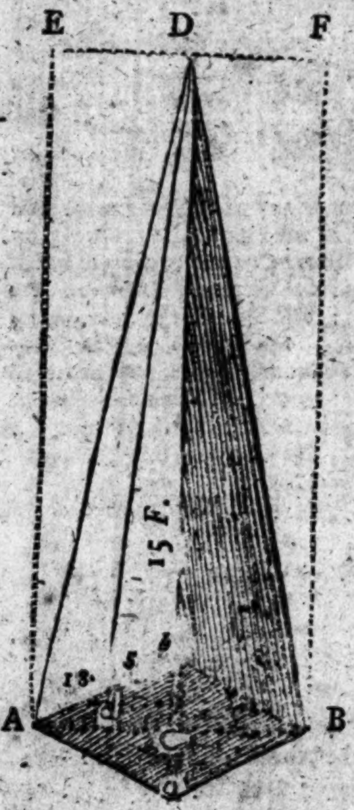
By Scale *and* Compaſſes.

Extend the Compaſſes from 144 to 180, that Extent will reach from *96* to 120 Feet: Then, to find the Area of the Baſe, extend the Compaſſes from 144 to 13.84, that Extent will reach from 48 to *4.6* Feet; add 120 Feet, and twice *4.6* Feet, and it makes 129.2 Feet, the ſuperficial Content, as before.

The Demonſtration of thoſe laſt Solids will be the ſame as in the firſt Section; for as in that, ſo in theſe, the Area of the Baſe is multiplied into the Length to find the Content, and the ſame Reaſon is given for one as for the other.

§ IV. *Of a* PYRAMID.

A Pyramid is a ſolid Figure, whoſe Baſe is a Polygon, and whoſe Sides are plain Triangles, their ſeveral Tops meeting together in one Point. To find the ſolid Content thereof, this is



The RULE.

Multiply the Area of the Baſe by a third Part of the Altitude, or Length, and the Product is the ſolid Content of the Pyramid.

Let ABD be a ſquare Pyramid, each Side of the Baſe being 18.5 Inches, and the Perpendicular Height CD is 15 Feet: Multiply 18.5 by 18.5, and the Product is 342.25, the Area of the Baſe in Inches; which multiplied by 5, a third Part of the Height, and the Product is 1711.25; this divided by 144, the Quotient is 11.88 Feet, the ſolid Content.

18.5
18.5

925
1480
185

342.25 Area of the Baſe.
5

144)1711.25(11.88 Content.

F.	I.	Pts.	
1	6	6	
1	6	6	
1	6	6	
	9	3	
		9	3
2	4	6	3
			5
11	10	7	3

By Scale *and* Compaſſes.

Extend the Compaſſes from 12 to 18.5 Inches, that Extent turned twice over from 5 Feet (a third Part of the Height) will fall at laſt upon 11.28 Feet, the ſolid Content.

To find the ſuperficial Content.

Multiply the ſlant Height (or Perpendicular of one of the Triangles) by half the Periphery of the Baſe 37, and the Product is 6668.88, which divided by 144, the Quotient is 46.31 Feet, the ſuperficial Content of all but the Baſe; then to that add 2.38 Feet, the Baſe, and it makes 48.69 Feet, the whole ſuperficial Content.

```
   180.24 The ſlant Height *d* D.
       37
   ------                                144)342.25(2.38
   126168                                    288
   54072                                     ---
   ------                                    542
144)6668.88(46.31                            432
    576      2.38                            ---
    ---      ----                            1105
    908     48.69 The whole Content.         1152
    864
    ---
     448
     432
     ---
     168
     144
     ---
      24
```

By Scale *and* Compaſſes.

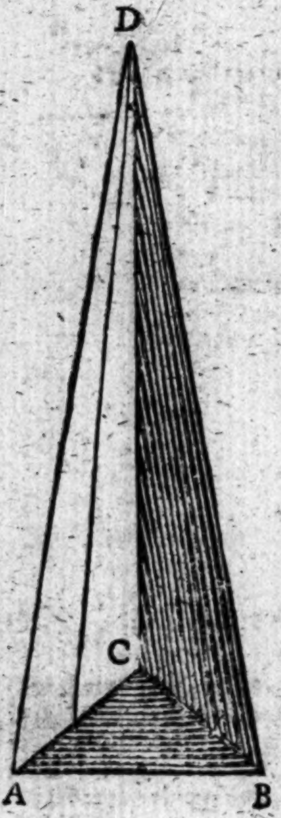
Extend the Compaſſes from 144 to 180.24 that Extent will reach from 37 to 46.31 Feet, the Area of the four Triangles; and extend the Compaſſes from 144 to 18.5 (one Side of the Baſe) that Extent will reach from 18.5 to 2.38 *ferè*; which added to the other, the Sum is 48.69, the whole Superficies.

DEMONSTRATION.

Every Pyramid is a third Part of the Priſm, that hath the ſame Baſe and Height, (by *Euclid*, 14. 7.)

That is, the ſolid Content of the Pyramid ABD (in the laſt Figure) is one third Part of its circumſcribing Priſm ABEF.

For every Pyramid that hath a ſquare Baſe (ſuch as A*a*B*b* in the laſt Figure) is conſtituted of an infinite Series of Squares, whoſe Sides or Roots are continually increaſing in arithmetical Progreſſion, beginning at the Vertex or Point D, its Baſe A*a*B*b* being the greateſt Term, and Its perpendicular Height CD is the Number of all the Terms: But the laſt Term multiplied into the Number of Terms, the Product will be triple the Sum of all the Series, (by *Lemma* III.) conſequently $\frac{NLL}{3}=S$. And S is equal to the ſolid Content of the Pyramid. From hence it will be eaſy to conceive, that every Pyramid is $\frac{1}{3}$ of its circumſcribing Priſm, (that is, of a Priſm of equal Baſe and Altitude) what Form ſoever its Baſe is of, *viz.* whether it be ſquare, triangular, pentangular, *&c.* You may very eaſily prove a triangular Pyramid to be a third Part of a Priſm of equal Baſe and Altitude, by cutting a triangular Priſm of Cork, and then cut that Priſm into three Pyramids, by cutting diagonally, as I have ſeveral times done, to ſatisfy myſelf and others.



Let ABCD be a triangular Pyramid, each Side of the Baſe being 21.5 Inches, and its perpendicular Height 16 Feet; the Gontent ſolid and ſuperficial is required.

Firſt, find the Area of the Baſe, by multiplying half the Side by the Perpendicular let fall from the Angle of the Baſe to the oppoſite Side; which Perpendicular will be found to be 18.62; the Half thereof is 9.31, which multiplied by 21.5, the Product is 200.165 Inches, the Area of the Baſe. Then, becauſe the Altitude 16 cannot exactly be divided by 3, therefore I take the third Part of 200.165, which is 66.72, and multiply it by 16, and the Product is 1067.52, which divided by 144, the Quotient is 7.41 Feet, the ſolid Content.

		F.	I.	Pts.	
9.31 Half the Perpendicular.					
21.5 The Side.	Side.	1	9	6	
	Half Perpendic.		9	4	
4655					
931		1	4	1	6
1862				7	2
3)200.165 Area Baſe.	Area Baſe.	1	4	8	8
					4
66.72 A third Part,					
16 Height.		5	6	10	8
					4
40032					
6672	3)	22	3	6	8
144)1067.52(7.41 Solid Content.	Content.	7	5	2	2
1008					
595					
576					
192					
144					
48					

In caſting this up by Feet and Inches, inſtead of multiplying by 16, the Height, I break 16 into two ſuch Numbers, as being multiplied together the Product may be 16, *viz.* into 4 and 4, and multiply firſt by one, and then the other; a third Part of the laſt Product is the Content.

By Scale *and* Compaſſes.

Firſt, find a geometrical mean Proportional, (as before directed) by dividing the Space between 21.5 and 9.31 into two equal Parts, and you will find the middle Point at 14.15, which is the mean Proportional ſought. Then extend the Compaſſes from 12 to 14.15, that Extent (turned twice over from 16 Feet, will fall at laſt upon 22.23; a third Part thereof is 7.41 Feet, the Content.

To find the ſuperficial Content.

Multiply the ſlant Height (or Perpendicular of one of the Triangles) by half the Periphery of the Baſe, and to that Product add the Area of the Baſe, the Sum is the whole ſuperficial Content.

192.1 Inches, the ſlant Height *d*D.
Half Periphery. 32.25=21.5+10.75.

9605
3842
3842
5763

6195.225 Inches, the Area of all but the Baſe.
200.165 Area of the Baſe add.

144)6395.390(44.42 Feet, the whole Content.
576

635
576

593
576

179
144

35

By Scale *and* Compaſſes.

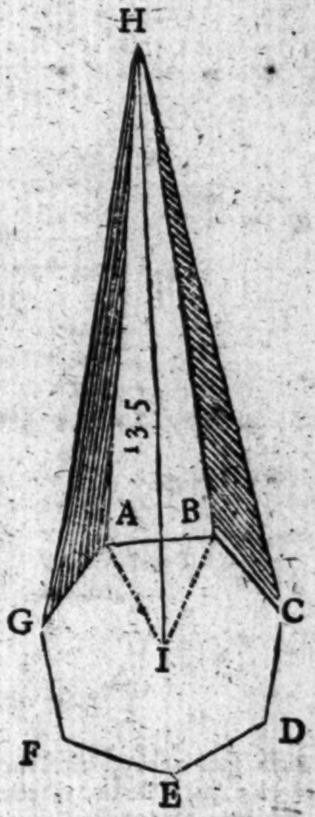
Extend the Compaſſes from 144 to 192.1, that Extent will reach from 32.25 (half the Periphery of the Baſe) to 43.02 Feet, the Content of the upper Part.

And extend the Compaſſes from 144 to half the Perpendicular 9.31, that Extent will reach from the Side 21.5 to 1.39 Feet, the Area of the Baſe, which added to the other makes 44.41 Feet, the Content of the Whole.

Let ABCDEFGH be a Pyramid, whoſe Baſe is a Heptagon, each Side thereof being 15 Inches, and the Perpendicular of the Heptagon is 15.58 Inches, and the perpendicular Height of the Pyramid HI is 13 5 Feet; the Content ſolid and ſuperficial is required.

Multiply 15.58 (the Perpendicular) by 52.5, half the Sum of the Sides of the Heptagon, and the Product is 817.95, which multiplied by 4.5, *viz.* $\frac{1}{3}$ of the Height, and the Product is 3680.775.

Then divide this laſt Product by 144, and the Quotient is 25.56 Feet, the Content.



15.58 The Heptagon's Perpendicular.
52.5 The half Sum of the Sides.

7790
3116
7790

817.950
4.5 A third Part of the Height.

4089750
3271800

144) 3680.7750(25.56 Solid Feet.
288

800
720

807
720

877
864

13

By Scale and Compaſſes.

Firſt, find a geometrical mean Proportional between 15.58 and 52.5 (as is before directed) which you will find to be 28.06; then extend the Compaſſes from 12 to 28.06, that Extent will reach from 4.5 (twice turn'd over) to 25.56 Feet.

To find the ſuperficial Content.

Multiply the Height taken from the Middle of one of the Sides of the Baſe 162.75 Inches, by the half Sum of the Sides 52.5 Inches, and the Product is 8544.375; which divided by 144, the Quotient is 59.335 Feet, the Content of the upper Part.

162.75

162.75
52.5
81375
32550
81375

144)8544.375(59.335 Feet.
1344
483
517
855
135

5.68 Baſe add.
65.015 The whole Content.

144)817.95(5.68
979
1155
3

By Scale *and* Compaſſes.

Extend the Compaſſes from 144 to 162.75, that Extent will reach from 52.5 to 59.335 Feet.

And extend the Compaſſes from 144 to 15.58, the Perpendicular of the Heptagon, the Extent will reach from 52.5 to 5.68 Feet, the Content of the Baſe; which add to the former, the Sum is 65.015, the whole ſuperficial Content.

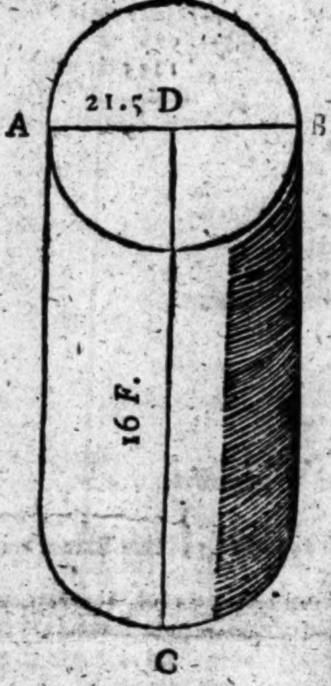
§ V. *Of a* CYLINDER.

A CYLINDER is a round Solid, having its Baſes circular, equal and parallel, in Form of a Rolling-Stone uſed in Gardens. To find the ſolid Content thereof, this is

The RULE.

Multiply the Area of the Baſe by the Length, and the Product is the ſolid Content.

Let



Let ABC be a Cylinder, whoſe Diameter AB is 21.5 Inches, and the Length CD is 16 Foot, the ſolid Content is required.

Firſt, ſquare the Diameter 21.5, and it makes 462.25; which multiply by .7854, and the Product is 363.05115. Then multiply this by 16, and the Product is 5808.8164. Divide this laſt Product by 144, and the Quotient is 40.34 Feet, the ſolid Content.

By Scale *and* Compaſſes.

Extend the Compaſſes from 13.54 to 21.5, the Diameter, that Extent (turned twice over from 16, the Length) will at laſt fall upon 40.34, the ſolid Content.

To find the ſuperficial Content.

Firſt, (by Chap. 1. Sect. IX. Prob. 2.) find the Circumference of the Baſe 67.54, which multiply by 16, the Product is 1080.64; which divided by 12, the Quotient is 90.05 Feet, the Curve Surface; to which add 5.04 Feet, the Sum of the two Baſes, and the Sum is 95.09 Feet, the whole ſuperficial Content.

```
    67.54                                363.05
       16                                     2
  -------                           -----------
    40524                          144)726.10(5.04
    6754                              ---------
  -------        90.05 }                  610
12)1080.64        5.04 } add.            -----
  -------        -----                     34
    90.05        95.09
```

By

By Scale *and* Compaſſes.

Extend the Compaſſes from 12 to 67.54, (the Circumference) that Extent will reach from 16, the Length, to 90.05 Feet, the Curve-Surface.

And extend the Compaſſes from 12 to 21.5, (the Diameter) that Extent (turned twice from .7854) will at laſt fall upon 2.52 Feet, the Area of one Baſe; which doubled is 5.04; this added to the Curve-Surface makes 95.09 Feet, the whole ſuperficial Content.

DEMONSTRATION.

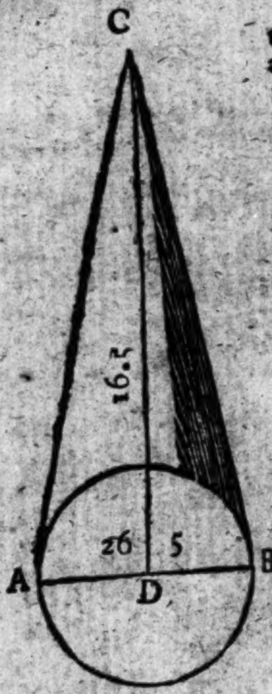
The ſolid Content of every Cylinder is found by multiplying the Area of its Baſe into its Height, as aforeſaid: For every right Cylinder is only a round Priſm, being conſtituted of an infinite Series of equal Circles; that of its Baſe, or End, being one of the Terms, and its Height CD (in the former Figure) is the Number of all the Terms: Therefore the Area of its Baſe AB being multiplied into CD, will be its Solidity (by *Lemma* I.) Let $D = AB$. and $H = CD$.
Then $.7854\,DD \times H =$ its Solidity.

§ VI. *Of a* CONE.

A CONE is a Solid, having a circular Baſe, and growing ſmaller and ſmaller till it ends in a Point, which is called the Vertex, and may be nearly repreſented by a Sugar-Loaf. To find the Solidity thereof, this is

The RULE.

Multiply the Area of the Baſe by a third Part of the perpendicular Height, and the Product is the ſolid Content.



Let ABC be a Cone, the Diameter of whoſe Baſe AB is 26.5 Inches, and the Height of the Cone DC is 16.5 Feet. Firſt, ſquare the Diameter 26.5, and it is 702.25; which multiply by .7854, and the Product is 551.54715; which multiply by 5.5, and the Product is 3033.47825; which divided by 144, the Quotient is 21.07 *ferè*, the ſolid Content of the Cone.

26.5 The Diameter.
26.5

1325
1590
530

702.55 The Square.
.7854

280900
351125
561800
491575

551.54|715 Area of the Baſe.
5.5 A 3d Part of the Height.

275773
275773

144)3033.503(21.07 Feet, the Content.

153
947

By Scale *and* Compaſſes.

Extend the Compaſſes from 13.54 to 26.5, (the Diameter) that Extent turned twice over from 5.5 (a third Part of the Height) will at laſt fall upon 21.07 Feet, the Content.

To

To find the ſuperficial Content.

Multiply half the Circumference 41.626 by the ſlant Height AC 198.46, the Product is 8261.09596; which divided by 144, the Quotient is 57.37, *ferè*, the Curve-Surface; to which add the Baſe, the Sum is 61.2, the ſuperficial Content.

41.626 The half Circumference of the Baſe.
198.46 Inches, the ſlant Height.

249756
166504
333008
374634
41626

144)8261.09596(57.37 Feet, *ferè*.
1061
530
989
144)551.54(383
1195
434
2

3.83 The Baſe add.

61.20 The whole Content.

By Scale *and* Compaſſes.

Extend the Compaſſes from 144 to 198.46, that Extent will reach from 41.626 to 57.37 Feet, the Curve-Surface.

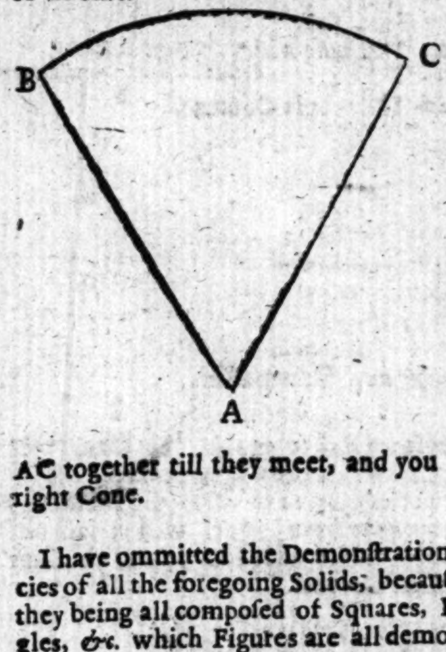
And extend the Compaſſes from 12 to 26.5, (the Diameter) that Extent, turned twice over from .7854, will at laſt fall upon 3.83 Feet, the Baſe; which added to 57.37, the Sum is 61.2 Feet, the ſuperficial Content.

DEMONSTRATION.

Every Cone is the third Part of a Cylinder of equal Baſe and Altitude. The Truth of this may eaſily be conceived by only conſidering, that a Cone is but a round Pyramid, and therefore it muſt needs have the ſame Ratio to its circumſcribing Cylinder, as the ſquare Pyramid hath to its circumſcribing Parallelopipedon, *viz.* as 1 to 3. However, to make it yet clearer, let it be farther conſidered, That

Every right Cone is conſtituted of an infinite Series of Circles, whoſe Diameters do continually increaſe in arithmetical Progreſſion, beginning at the Vertex, or Point C, the Area of its Baſe AB being the greateſt Term, and its perpendicular Height DC the Number of all the Terms; therefore the Area of the Circle of the Baſe, multiplied by a third Part of the Altitude DC, will be the Sum of all the Series, equal to the Solidity of the Cone, by *Lemma* III.

The Curve-Superficies of every right Cone, is equal to half the Rectangle of the Circumference of its Baſe into the Length of its Side.



For the Curve-Surface of every right Cone is equal to the Sector of a Circle, whoſe Arch BC is equal to the Periphery of the Baſe of the Cone, and Radius AB equal to the ſlant Side of the Cone. Which will appear very evident, if you cut a Piece of Paper in the Form of a Sector of a Circle, as A B C, and bend both the Sides AB and AC together till they meet, and you will find it to form a right Cone.

I have ommitted the Demonſtrations touching the Superficies of all the foregoing Solids, becauſe I thought it needleſs, they being all compoſed of Squares, Parallelograms, Triangles, *&c.* which Figures are all demonſtrated before. And if the Area of all ſuch Figures as compoſe the Solid be found ſeverally, and added together, the Sum will be the ſuperficial Content of the Solid.

§ VII. *Of the Fruſtum of a* PYRAMID.

A Fruſtum of a Pyramid, is the remaining Part when the Top is cut off by a Plane parallel to the Baſe. To find the ſolid Content thereof, there are ſeveral Rules.

RULE I.

To the Rectangle (or Product) of the Sides of the two Baſes add the Sum of their Squares; that Sum being multiplied into one third Part of the Fruſtum's Height, will give its Solidity, if the Baſes be ſquare.

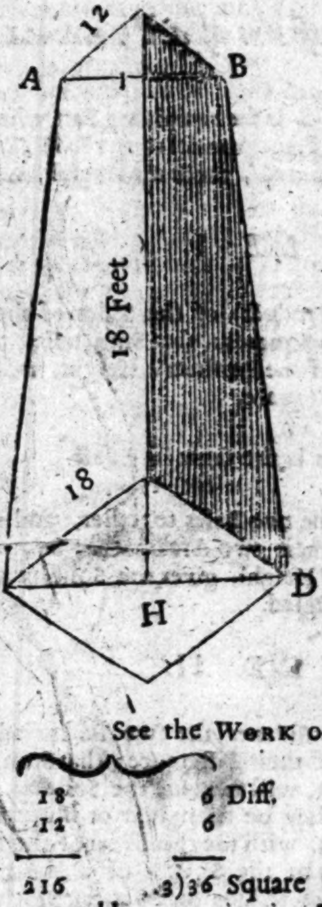
Or thus, which is the ſame in Effect.

Multiply the Areas of the two Baſes together, and to the ſquare Root thereof add their two Areas; that Sum multiplied by one Third of the Height, gives the Solidity of any Fruſtum, ſquare or multangled.

RULE II.

To the Rectangle of the Sides of the two Baſes, add one third Part of the Square of their Difference; that Sum being multiplied into the Height, will produce the Solidity, if the Baſes be Squares: But if they be triangular or multangular, the Rectangle of the Sides, with the third Part of the Square of their Difference, will be the Square of a mean Side; and the ſquare Root thereof will be ſuch a mean Side as will reduce the tapering Solid to a Priſm equal thereunto.

Example. Let ABCD be the Fruſtum of a ſquare Pyramid, the Side of the greater Baſe 18 Inches, and the Side of the leſſer 12 Inches, and the Height 18 Foot; the Solidity thereof is required,



Firſt, multiply the two Sides together, 18 by 12, and the Product is 216, and the Difference of the Sides is 6, whoſe Square is 36; a third Part thereof is 12, which added to 216, the Sum is 228 Inches, the Area of a mean Baſe; which multiplied by 18 Feet, the Length, the Product is 4104; this divided by 144, the Quotient is 28.5 Feet, the Content.

Or, by the firſt Rule, thus; the Square of 18 is 324, and the Square of 12 is 144, and the Rectangle of 18 by 12 is 216; the Sum of theſe three 684, which multiplied by 6, the Product is 4104; which divided by 144, the Quotient is 28.5 Feet, the ſame as before.

See the Work of both Ways.

18
12
———
216
12 add
———
228 the Sum
18 the Height
———
1824
228
———
144)4104(28.5
———
1224
720
———
...

6 Diff.
6
———
3)36 Square
———
12 a Third

18
18
———
324 Square

12
12
———
144 Square

324
144
216
———
684 the Sum
6 a 3d of the Height
———
144)4104(28.5 Feet
———
1224
720
———
...

(5)

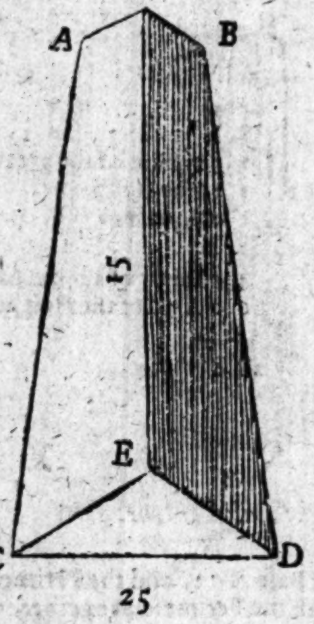
By Feet and Inches thus:

```
            F  I.      I.    }          {
Multiply    1  6       6     }          {
      by    1          6     }          {   F. I.
           ------    -----   }          {    2  3 Square of the greater
Product     1  6   3)36 q.   }          {    1  6 The Rectangle.
    add     0  1     ----    }          {    1  0 Square of the leſs.
           ------     12     } Or thus, {   -----
Multiply    1  7             }          {    4  9 Triple of a mean Area
      by   18  0 Height.     }          {    6  0 A 3d of the Height
           ------            }          {   -----
           18  0             }          {   28  6
            9  0             }          {
            1  6             }          {
           ------            }          {
Content    28  6             }          {
```

To find the ſuperficial Content.

The Perimeter of the greater Baſe is 72, and the Perimeter of the leſſer Baſe is 48; add both the Perimeters together, the Sum is 120; the Half thereof is 60; which multiplied by 18 Feet, the Product is 1080; this divided by 12, the Quotient is 90 Feet; to which add the two Baſes 2.25 Feet, and 1 Foot, the Sum is 93.25 Feet, the whole ſuperficial Content.

```
   18        12         18 The Height.
    4         4         60
  ----      ----      -----
   72        48    12)1080
   48                   90 Feet.
  ----                  2.25 The greater Baſe.
2)120                   1    The leſſer Baſe.
  ----                -----
   60                   93.25 Sum.
```



Again. Let ABCD be the Fruſtum of a triangular Pyramid, each Side of the greater Baſe 25 Inches, and each Side of the leſſer Baſe 9 Inches, and the Length 15 Feet; the ſolid Content thereof is requir'd.

By the ſecond Rule; multiply 25 by 9, and the Product is 225; and the Difference between 25 and 9 is 16; which ſquared makes 256, a third Part thereof is 85.333, which added to 225, and the Sum is 310.333; this multiplied by .433, the Product is 134.374, &c, which is the Area of a mean Baſe: and that multiplied by 15 Feet, the Length, the Product is 2015.610, which divided by 144, the Quotient is 13.99 Feet, the Solidity.

Or thus, by the latter Part of the firſt Rule: Find the Area of the greater Baſe, which you will find to be 270.625, and the Area of the leſſer Baſe will be 35.073; theſe two Areas multiplied together, the Product is 9491.630625, the ſquare Root thereof is 97.425, to which add the two Areas, and the the Sum is 403.123; which multiplied by a third Part of the Length 5, and the Product is 2015.610; and that divided by 144, and the Quotient is 13.99 Feet, as before.

See the Working of both.

```
         15     25
          9      9
        ----   ----
Product 225     16 Difference
                16
               ----
                96
               16
              -----
             3)256 the Square
              -----
               85.333 a 3d Part
              225. add
              -------
              310.333
                 .433 Tabular Number (vide p. 87)
              -------
              930999
             930999
           1241332
           ----------
          134.374189 mean Area.
                  15 Length.
          ------------
          671870945
         134374189
         ------------
    144)2015.61|2835(13.99 Feet.
         ------------
         575
         1436
          1401
         -----
          105
```

25

```
      25              9            .433
      25              9              81
    ----            ---          ------
     125           81 Square        433
     50                            3464
    ----                        -------
     625 Square.                 35.073
    .433
  ------
    1875                 270.625
   1875                   35.073
  2500                 ---------
  ------                  811875
  270.625 Area.          1894375
                        1353125
                        811875
                       ---------
                        . .
                        9491.630625(97.425
                        81
                       ----
                    187)1391
                        1309
                       ------
                     1944)8263
                          7776
                         ------
                     19482)48706
                           38964
                           -------
                    194845)974225
                           974225
                           -------
                           ......
```

270.625

270.625 Greater Area.
97.425 The mean Proportional.
35.073 The leſſer Area.

403.123 The Triple of a mean Area
5 A third Part of the Height.

144)2015.615(13.99 Feet, the Solidity.
575
1436
1401

105

In finding the Area of the triangular Baſe I multiplied by .433, becauſe that is the Area of the equilateral Triangle when the Side thereof is 1. A Table of the Areas, or Multipliers, for finding the Areas of Polygons, you'll find in Page 87.

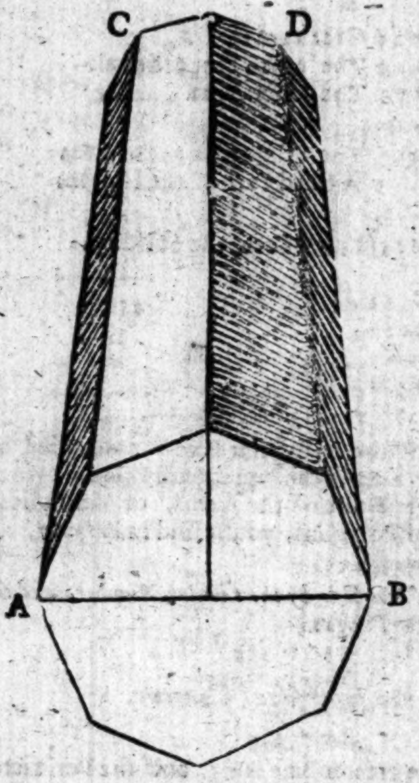
Multiply the Square of the Side by the tabular Number, and the Product is the Area of the Polygon.

To find the ſuperficial Content.

The Perimeter of the greater Baſe is 75, and the Perimeter of the leſſer Baſe is 27; the Sum of both is 102, and the half Sum is 51, which multiplied by 15 Feet, the Product is 765; which divided by 12, the Quotient is 63.75; to which add the Sum of the two Baſes 2.12 Feet, and the Sum is 65.87 Feet, the whole ſuperficial Content.

NOTE, That 51 ſhould have been multiplied by the ſlant Height, but the Difference it would make is but .06 of a Foot, which is inconſiderable.

Again.



Again. Suppoſe ABCD to be the Fruſtum of a Pyramid, having an Octagonal Baſe, each Side thereof being 9 Inches, and each Side of the leſſer Baſe 5 Inches, and the Length or Height 10.5 Feet; the Solidity is required.

By the ſecond Rule. Multiply the greater Side 9, by the leſſer Side 5, and the Product is 45; then the Difference between 9 and 5 is 4, which ſquared makes 16; a third Part thereof is 5.3333, which added to 45, the Sum is 50.3333; multiply this laſt by the Number in the Table, 4.8284, and the Product is 243.0292, the Area of a mean Baſe; which multiplied by the Height 10.5 Feet, and the Product is 2551.8066; then divide this laſt Product by 144, and the Quotient is 17.72 Feet, the ſolid Content.

See the WORK.

See the Work.

Mult. 9 Inches 9 from the greater Side.
by 5 Inches 5 ſubtract the leſſer.

Prod. 45 4
4

3)16 ſquar'd.

5.3333 a third Part.
Add 45

Sum 50.3333 the Square of a mean Side.
4818.4 Tabular Number, Page 87.

2013332
402666
10067
4026
201

243.0292 a mean Area.
10.5 the Height.

12151460
2430292

144)2551.80|660(17.72
144

1111
1108

1038
1008

300
288

12

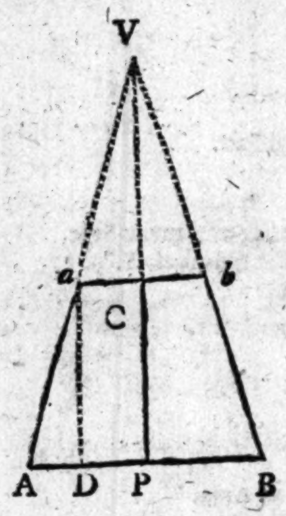
To find the ſuperficial Content.

The Perimeter of the greater Baſe is 72, and the Perimeter of the leſſer Baſe is 40, and their Sum is 112; the Half thereof is 56, which multiplied by the Height 10.5 Feet, and the Product is 588; which divided by 12, the Quotient is 49 Feet; to which add the Sum of the two Baſes 3.55, and the Sum is 52.55 Feet, the whole ſuperficial Content.

DEMON-

DEMONSTRATION.

From the Rules delivered in the fourth and ſixth Sections, the two foregoing Rules may eaſily be demonſtrated.



Suppoſe a ſquare Pyramid, as ABV, to be cut by a Plane at *a b*, parallel to its Baſe AB, and it were required to find the Solidity of the Fruſtum or Part *a b* A B. Let there be given

$D = BA$, the Side of the greater Baſe.
$d = ba$, the Side of the leſſer Baſe.
$H = CP$, the perpendicular Height.

First, 1 $D - d : H :: d : \frac{dH}{D-d} = VC$ by the Figure.
Then 2 $DD \times \frac{H + VC}{3} =$ the whole Pyramid BVA by Section the fourth.
And 3 $dd \times \frac{1}{3} VC =$ the Pyramid *a*V*b* cut off.

Then, in the 2d and 3d Steps, if inſtead of VC you take $\frac{dH}{D-d} =$ to it by the firſt Step, it will be, *viz.* 1.2 | 4 | $\frac{DDDH}{3D-3d}$ = the whole Pyramid BVA.

and 1.3 | 5 | $\frac{dddH}{3D-3d}$ = the Pyramid *a*V*b*.

4—5 | 6 | $\frac{DDDH-dddH}{3D-3d}$ = the Fruſtum *ab*AB.

And by dividing $DDD-ddd$ by $D-d$, and then multiplying the Quotient by $\frac{1}{3}H$, the laſt Step will be reduced to $DD+Dd+dd : \times \frac{1}{3}H$ = the Fruſtum *ab*AB; which in Words is thus:

To the Rectangle of the Sides of the two Baſes, add the Sum of their Squares; that Sum being multiplied into one Third of the Fruſtum's Height, will give its Solidity, which is the ſame as the firſt Rule of this Section.

See the Work of the Diviſion.

$$\begin{array}{l} D-d)\,DDD-ddd\,(DD+Dd+dd \\ \qquad DDD-DDd \\ \hline \qquad\qquad DDd-ddd \\ \qquad\qquad DDd-Ddd \\ \hline \qquad\qquad\qquad Ddd-ddd \\ \qquad\qquad\qquad Ddd-ddd \\ \hline \qquad\qquad\qquad\quad 0 \quad 0 \end{array}$$

The ſame Reaſon will hold good for all Fruſtums of Pyramids or Cones, whether the Baſe be triangular or multangular, becauſe the Squares of the Sides of any Figure, or the Squares of the Diameters of Circles, are proportional to the Area, which proves the latter Part of the ſaid firſt Rule.

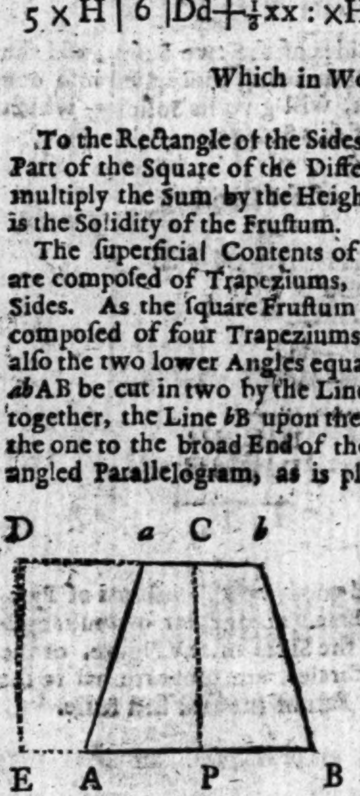
Again, to prove the ſecond Rule.

Suppoſe	1	$x = D - d$. And $F =$ the Fruſtum.
then	2	$DD + Dd + dd = \frac{3F}{H}$ by the laſt.
1 ⊚ 2	3	$xx = DD - 2Dd + dd$.
$2 - 3$	4	$3Dd = \frac{3F}{H} - xx$.
$\overline{4 \div 3}$	5	$Dd = \frac{F}{H} \frac{4}{3}xx$. Or $Dd + \frac{1}{3}xx = \frac{F}{H}$
$5 \times H$	6	$Dd + \frac{1}{3}xx : \times H = F$, the Fruſtum *ab*AB.

Which in Words is thus:

To the Rectangle of the Sides of the two Baſes add one third Part of the Square of the Difference of the ſaid Sides, and multiply the Sum by the Height of the Fruſtum, the Product is the Solidity of the Fruſtum.

The ſuperficial Contents of Fruſtums (all but the Baſes) are compoſed of Trapeziums, ſo many as the Fruſtum has Sides. As the ſquare Fruſtum *ab*AB, in the laſt Figure, is compoſed of four Trapeziums, having the two upper and alſo the two lower Angles equal; if therefore the Trapezium *ab*AB be cut in two by the Line CP, and the two Pieces laid together, the Line *b*B upon the Line *a*A, the narrow End of the one to the broad End of the other, it will form a right-angled Parallelogram, as is plain by the Figure annexed; the Parallelogram DCEP being equal to the Trapezium *ab*AB; becauſe the Side D*a* is equal to PB, and EA is equal to *a*C. Therefore, to find the Area of the Trapezium, add half the Side *ab* to half the Side AB, and it makes DC or EP; which multiply by the Height PC, the Product is the Area of the Parallelogram DCEP, equal to the Trapezium *ab*AB; then, if that be multiplied by the Number of Trapeziums, the Product will be the ſuperficial



cial Content of the Fruſtum, wanting the Baſes. Or if the whole Perimeter of the greater Baſe be added to the Perimeter of the leſſer Baſe, and half the Sum multiplied by the Height, the Product will be the ſuperficial Content of all the Trapeziums at once.

NOTE, That half the Sum of the Perimeters ſhould be multiplied by the ſlant Height, up the Middle of one of the Trapeziums; but in the foregoing *Examples* I have multiplied by the perpendicular Height, becauſe the Difference is very inconſiderable.

§ VIII. *Of the Fruſtum of a* CONE.

A Fruſtum of a Cone is that Part which remains, when the Top End is cut off by a Plane parallel to the Baſe. To find the ſolid Content, the Rules are the ſame in Effect as for the Fruſtum of a Pyramid.

RULE I.

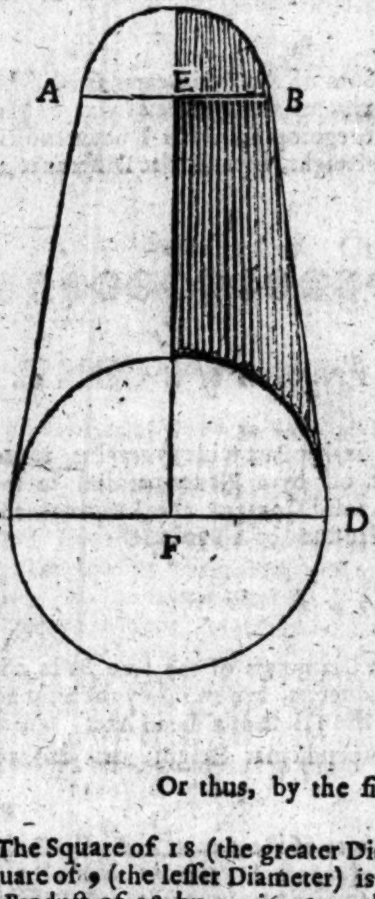
To the Rectangle of the Diameters of the two Baſes add the Squares of the ſaid Diameters, and multiply the Sum by .7854, the Product will be the Triple of a mean Area, which multiplied by $\frac{1}{3}$ of the perpendicular Height, that Product will be the ſolid Content.

Or thus: Multiply the Areas of the greater and leſſer Baſes together, and out of the Product extract the ſquare Root, and add the two Areas and ſquare Root together, and multiply the Sum by one Third of the perpendicular Height, the Product is the ſolid Content.

RULE II.

To the Rectangle of the greater and leſſer Diameters, add one third Part of the Square of their Difference, and multiply the Sum by .7854, the Product is a mean Area;

which multiplied by the perpendicular Height, the Product is the Solidity.



Example. Let ABCD be the Fruſtum of a Cone, whoſe greater Diameter CD is 18 Inches, and the leſſer Diameter AB 9 Inches, and the Length 14.25 Feet; the ſolid Content is required.

Multiply 18 by 9, and the Product is 162, and the Difference between 18 and 9 is 9, whoſe Square is 81, a third Part is 27; which add to 162, the Sum is 189; this multiplied by .7854, the Product is 148.44; which divided by 144, the Quotient is 1.03 Feet, the Area of a mean Baſe; which multiply by 14.25 Feet, the Height, the Product is 14.6775 Feet, the ſolid Content.

Or thus, by the firſt Rule.

The Square of 18 (the greater Diameter) is 324, and the Square of 9 (the leſſer Diameter) is 81, and the Rectangle, or Product of 18 by 9, is 162; the Sum of theſe three is 567, which multiplied by .7854, the Product is 445.3218; which divided by 144, the Quotient is 3.09 Feet, the triple Area of a mean Baſe; this multiplied by 4.75 Feet (a third Part of the Height) and the Product is 14.6775 Feet, the Solidity, the ſame as before.

See the Work.

18
9
162
Add 27
Sum 189

18 from
9 ſubtract
9 remains
9
3)81 Square.
27 a Third.

.7854
189
70686
62832
7854
144)148.44|06(1.03
144
444
432
12

Height 14.25 Feet
Area Baſe 1.03 Feet
4275
1425
Solid Content 14.6 75 Feet

324 The Square of 18
162 The Rectangle.
81 The Square of 9.
567 The triple Square of a mean Diameter.

.7854
567
54978
47124
39270
144)445.32|18 (3.09
1332
36

4.75 a Third of the Height.
1545
2163
1236
The Solidity 14.6775

To find the ſuperficial Content.

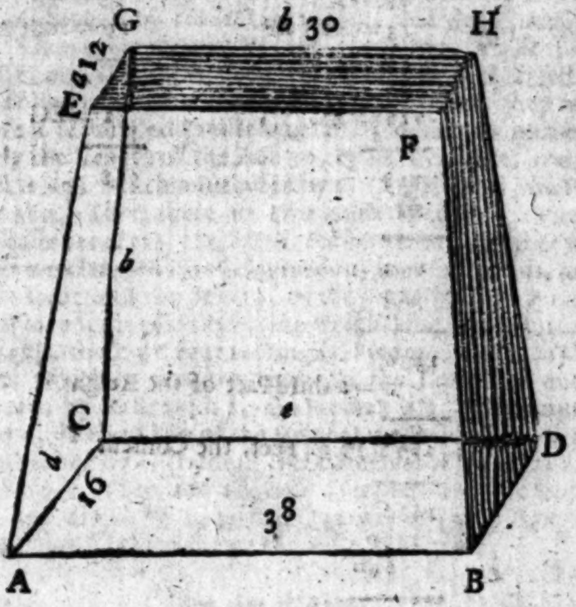
By Chap. I. Sect. IX. *Problem* 2. you will find the Circumference of the greater Baſe to be 56.5488, and of the leſſer Baſe 28.2744; the Sum of both is 84.8232; the half Sum is 42.4116; which multiplied by 14.25 Feet, and the Product is 604.36, *&c.* which divided by 12, the Quotient is 50.36 Feet, the Curve-Surface; to which add the Sum of the two Baſes, 2.21 Feet, the Sum is 52.57 Feet, the whole ſuperficial Content.

§ IX. *To meaſure the* Fruſtum *of a rectangled* Pyramid, *called a* PRISMOID, *whoſe Baſes are parallel to one another, but diſproportional.*

The RULE.

TO the greateſt Length add half the leſſer Length, and multiply the Sum by the Breadth of the greater Baſe, and reſerve the Product.

Then, to the leſſer Length add half the greater Length, and multiply the Sum by the Breadth of the leſſer Baſe, and add this Product to the other Product reſerved, and multiply that Sum by a third Part of the Height, and the Product is the ſolid Content.



Example. Let A B C D E F G H be a Priſmoid given, the Length of the greater Baſe AB 38 Inches, and its Breadth AC 16 Inches; and the Length of the leſſer Baſe EF is 30 Inches, and its Breadth 12 Inches, and the Height 6 Feet; the ſolid Content is required.

To the greater Length AB 38, add half EF the leſſer Length 15, the Sum is 53; which multiply by 16, the greater Breadth, and the Product is 848, which reſerve.

Again. To EF 30, add half AB 19, and the Sum is 49; which multiply by 12, (the leſſer Breadth EG) the Product is 588; to which add 848, (the reſerved Product) and the Sum is 1436; which multiply by 2, (a third Part of the Height) and the Product is 2872; divide this Product by 144, and the Quotient is 19.94 Feet, the ſolid Content.

38=AB
15=Half EF

53
16=AC

318
53

848
588

1436
2=a third Part of the Height.

144)2872(19.94 Feet, the Content.

1432
1360
640

64

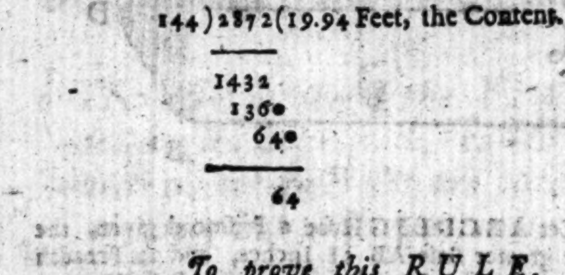
30=EF
19=Half AB

49
12=EG

588

To prove this RULE.

Let us ſuppoſe the Solid cut into Pieces, ſo as to make it capable of being meaſured by the foregoing Rules, thus: Let ABCD repreſent the greater Baſe, and EFGH the leſſer Baſe; and let the Solid be ſuppoſed to be cut thro' by the Lines *ac*, *bd*, and *ef*, *gh*, from the Top to the Bottom, ſo will there be a Parallelopipedon having its Baſes equal to the leſſer Baſe EFGH, and its Height 6 Feet equal to the Height of the Solid: Multiply 30 (the length of the Baſe)



Baſe) by 12, (the Breadth thereof) and the Product is 360; which multiplied by the Height 6 Feet, and the Product is 2160. Then there are two Wedge-like Pieces, whoſe Baſes are *ab*EF, and GH*cd*; if theſe two Pieces be laid together, the thick End of one to the thin End of the other, they will compoſe a rectangled Parallelopipedon; which to meaſure, multiply the Length of the Baſe 30, by its Breadth 2, and the Product is 60; which multiplied by 6, (the Height) the Product is 360. Then there are two other Wedge-like Pieces, whoſe Baſes are *e*EgG, and F*f*H*h*; theſe two laid together will compoſe a rectangled Parallelopipedon; to meaſure this, multiply the Length of the Baſe 12, by the Breadth 4, the Product is 48; which multiplied by 6, (the Height) the Product is 288. And, *laſtly*, there are four rectangled Pyramids, at each Corner one; which to meaſure, multiply the Length of one of the Baſes 4, by its Breadth 2, the Product is 8; which multiplied by 2, (a third Part of the Height) the Product is 16, and that multiplied by 4, (becauſe there are four of them) and the Product is 64. Then add all theſe together, and the Sum is 2872; and divide by 144, the Quotient is 19.94 Feet, the ſame as before; which ſhews the Rule to be true.

See the Work.

12	30	12	4
30	2	4	2
360	60	48	8
6	6	6	2
2160	360	288	16
360			4
288			
64			64

144)2872(19.94 Feet, the whole Content.

1432
1360
640
64

Th

To find the ſuperficial Content.

Half the Perimeter of the greater Baſe is 54, and half the Perimeter of the leſſer Baſe is 42; which added together the Sum is 96, which multiplied by 6, (the Height) the Product is 576; divide this Product by 12, the Quotient is 48 Feet; to which add the Sum of the two Baſes 6.72 Feet, and the Sum is 54.72 Feet, the whole ſuperficial Content.

* * *

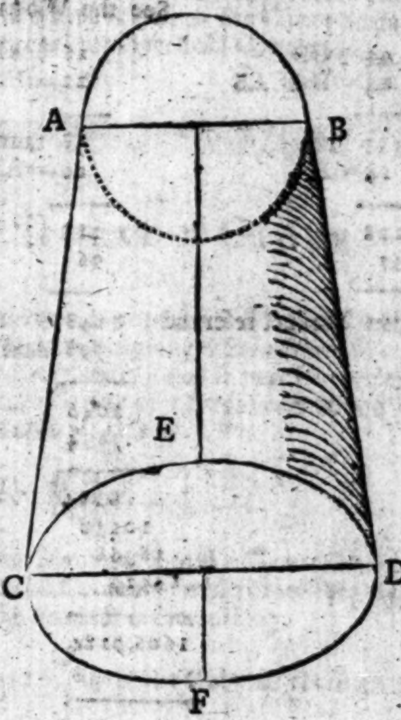
§ X. *To meaſure a* CYLINDROID; *that is, a* Fruſtum *of a* CONE *having its Baſes parallel to each other, but unlike.*

The RULE.

TO the longeſt Diameter of the greater Baſe add half the longeſt Diameter of the leſſer Baſe, and multiply the Sum by the ſhorteſt Diameter of the greater Baſe, and reſerve the Product.

Then, to the longeſt Diameter of the leſſer Baſe add half the longeſt Diameter of the greater Baſe, and multiply the Sum by the ſhorteſt Diameter of the leſſer Baſe, and add the Product to the former reſerved Sum, and that Sum will be the triple Square of a mean Diameter; which multiplied by .7854, and that Product multiplied by a third Part of the Height, the Product is the ſolid Content.

Example. Let AB CD be a Cylindroid, whoſe bottom Baſe is an Oval, the tranſverſe Diameter being 44 Inches, and the conjugate Diameter 14 Inches; and the upper Baſe is a Circle whoſe Diameter is 26 Inches, and the Height of the Fruſtum is 9 Feet; the Solidity is required.



To 44 (the greater Diameter of the lower Baſe) add 13, half the Diameter of the leſſer Baſe, the Sum is 57; which multiplied by 14, (the conjugate Diameter of the greater Baſe) the Product is 798, which reſerve. Then to 26 (the Diameter of the leſſer Baſe) add 22, (half the tranſverſe Diameter of the greater Baſe) and the Sum is 48; which multiplied by 26, (the Diameter of the leſſer Baſe) the Product is 1248; to which add the former reſerved Product, the Sum is 2046; which multiplied by .7854, and the Product is 1606.9284; which multiplied by 3, (a third Part of the Height) the Product is 4820.7852; which divided by 144, the Quotient is 33.47 Feet, the ſolid Content.

See the Work.

44=CD	26=AB
13=Half AB	22=Half CD
57 Sum	48 Sum
14=EF	26=AB
228	288
57	96
798 Product reſerved	1248
	798 Add

2046
.7854

8184
10230
16368
14322

1606.9284
3

144)4820.78|52(33.47

500
687
1118

110

This Rule being the ſame with that in the laſt Section, the Proof of that may ſerve as a ſufficient Proof of this, if what has been before written be well conſidered.

To find the ſuperficial Content.

To the Periphery of the Ellipſis 97.41 add the Periphery of the Circle 81.68, and the Sum is 179.09; the Half thereof 89.545, multiplied by 9, the Product is 805.905; which divided by 12, the Quotient is 67.16 Feet, the Curve-Surface:

Then the Area of the Ellipſis is 3.36 Feet, and the Area of the Circle is 3,68 Feet; both which added to the Curve Surface, the Sum is 74.2 Feet, the whole ſuperficial Content.

§ XI. *Of a* SPHERE *or* GLOBE.

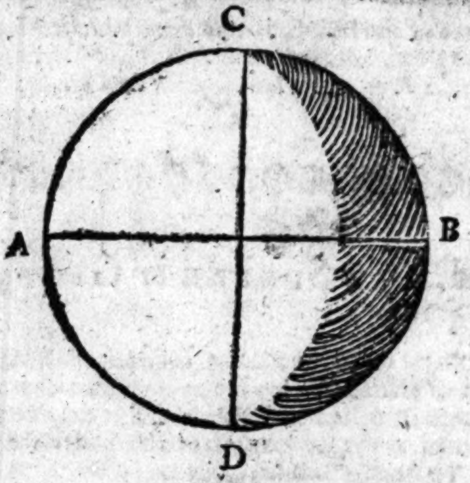
A SPHERE, or GLOBE, is a round ſolid Body, every Part of whoſe Surface is equally diſtant from a Point within it, called its Center; and it may be conceived to be formed by the Revolution of a Semicircle round its Diameter. To find its Solidity, this is

The RULE.

1. Multiply the Axis or Diameter into the Circumference, the Product is the ſuperficial Content; which multiply by a ſixth Part of the Axis, the Product is the Solidity.

2. Or thus; As 21 : to 11 :: ſo is the Cube of the Axis to the ſolid Content.

3. Or, As 1 ; is to .5236 ; ; ſo is the Cube of the Axis to the ſolid Content.



Example. Let ABCD be a Globe whoſe Axis is 20 Inches, then the Circumference will be 62.832: Then, by the firſt Rule, multiply the Circumference by the Axis, and the Product will be 1256.64, which is the ſuperficial Content in Inches; take a ſixth Part thereof, which is 209.44, (becauſe an exact ſixth Part of 20 cannot be taken) multiply that ſixth Part by 20, (the Axis) and the Product is 4188.8, the Solidity in Inches. Or, if you multiply the ſuperficial Content by the Axis, and take a ſixth Part of the Product, he Anſwer will be the ſame.

Or thus, by the ſecond Rule:

The Cube of the Axis is 8000, which multiplied by 11, the Product is 88000: which divided by 21, the Quotient is 4190.47, the Solidity.

Or, by the third Rule:

If the Cube of the Axis be multiplied by .5236, the Product is 4188.8 the Solidity, the ſame as by the firſt Way. If you divide 4188.8 by 1728, the Quotient is 2.424 Feet.

See the Work.

62.832
20

6)1256.640 The ſuperficial Content.

209.44 A ſixth Part.
20

4188.80 The Solidity in Inches.

As 21 : 11 :: 8000
11
21)88000(4190.47 The Content

40
190
100
160

13

As 1 : .5236 :: 8000
8000
1728)4188.8000(2.424 Feet, the Solidity.

7328
4160
7040

128

NOTE, If the Axis of a Globe be 1, the Solidity will be .5236; and if the Circumference be 1, the Solidity will be .016887.

By Scale *and* Compaſſes.

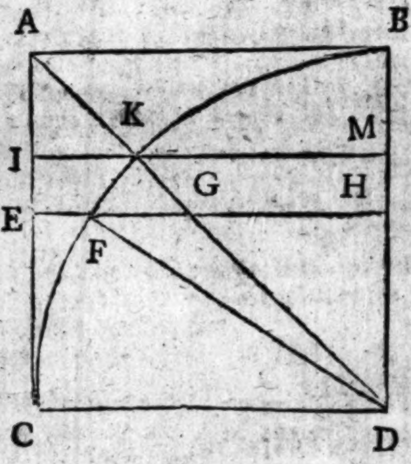
Extend the Compaſſes from 1 to 20, (the Axis) that Extent turned three times over from .5236, will at the laſt fall upon 4188,8, the ſolid Content in Inches Or, Extend the Compaſſes from 1728 to 8000, (the Cube of the Axis) that Extent will reach from .5236 to 2.424, the ſolid Content in Feet.

Extend the Compaſſes from 1 to 20, (the Axis) that Extent, turned twice over from 3.1416, will at laſt fall upon 1256.64, the ſuperficial Content in Inches. Or, Extend the Compaſſes from 144 to 400, (the Square of the Axis) that Extent will reach from 3.1416 to 8.72, the ſuperficial Content in Feet.

DEMONSTRATION.

Every Sphere is equal to a Cone whoſe perpendicular Axis is the Radius of the Sphere, and its Baſe a Plane, equal to all the Surface of it.

For you may conceive the Sphere to conſiſt of an infinite Number of Cones, whoſe Baſes, taken all together, compoſe the Surface, and whoſe Vertexes meet all together in the Center of the Sphere: Hence the Solidity of the Sphere will be gained by multiplying its Surface by $\frac{1}{3}$ of its Radius.



Let the Square ABCD, the Quadrant CBD, and the right-angled Triangle ABD, be ſuppoſed all three to revolve round the Line BD as an Axis; then will the Square generate a Cylinder, the Quadrant an Hemiſphere, and the Triangle a Cone, all of the ſame Baſe and Altitude.

Then the Square of $EH\,(=\square FD)=\square FH+\square DH$ (but $DH=GH$.) And ſince Circles are as the Squares of their Diameters, by *Euclid*, 12. 2.) the Circle made by the Revolution of EH muſt be equal to both the Circles made by the Motions of FH and GH.

If you take the Circle made by the Revolution of FH from both, there will remain the Circle made by the Motion of GH, equal to the Ring deſcribed by the Motion of EF. And thus it will always be, where-ever you draw the Line EH or IM, *&c.*

Therefore the Aggregate, or Sum, of all the Rings made by the Revolution of EF's, muſt be equal to that of all the Circles made by the Motion of GH's, *i. e.* the Diſh like Solid, formed by the revolving Rings, will be equal to the Cone, formed by the Revolution of the GH's, which are the Elements of the Triangle ABD; that is, the Diſh like Solid will be as the Cone, $\frac{1}{3}$ of the circumſcribing Cylinder, and conſequently the Hemiſphere muſt be $\frac{2}{3}$ of it: Wherefore the Sphere is $\frac{2}{3}$ of the circumſcribing Cylinder.

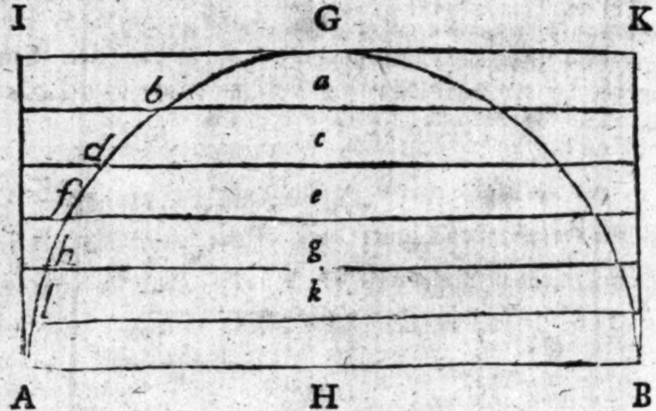
Let the Radius of the Sphere be $r = CD$, then the Diameter will be $2r$; let the Surface of the Sphere, generated by the revolving Semicircle, be called S, and that of the Cylinder formed by the Revolution of $2\,AC = 2r =$ Diameter, be called ſ. Wherefore, in what was juſt now proved, the Expreſſion for the Solidity of the Sphere in this Notation will be $\frac{rS}{3}$, and putting c equal to the Circumference of the Baſe, or for the Periphery of a great Circle of the Sphere, the Curve-Surface of the Cylinder will be $2rc$; alſo $\frac{rc}{2}$ will be the Area of a great Circle, by Sect. IX. of Chap. I. *Problem* 1.) and this multiplied by $2r$, makes rrc, which is the Solidity of the Cylinder, by Sect. V. of this Chapter. Now, ſince ſ was put equal to $2rc =$ to the Curve-Surface of the Cylinder $\frac{\text{ſ}r}{2}$, (by ſubſtituting ſ for $2rc$) will be alſo = to the Solidity of the Cylinder. Now, ſince the Sphere is $= \frac{2}{3}$ of the Cylinder, $\frac{rS}{3} = \frac{2}{3}\,\frac{\text{ſ}r}{2}$; that is, $\frac{rS}{3} = \frac{2\text{ſ}r}{6} = \frac{\text{ſ}r}{3}$.

Wherefore $rS = r\text{ſ}$, that is, dividing by r, $S = \text{ſ}$; or the Surface of the Sphere is equal to the Curve-Surface of the Cylinder, but the Curve Surface of the Cylinder was $2rc$.

Wherefore, to find the Area of the Surface of either Sphere or Cylinder. you muſt multiply the Diameter ($= 2r$) by the Circumference of a great Circle of the Sphere, or by the Periphery of the Baſe. From this Notation alſo $\frac{rc}{2}$, the Area of a great Circle of the Sphere, is plainly $\frac{1}{4}$ of 2rc, the Surface of the Sphere; that is, the Surface of the Sphere is quadruple of the Area of the greateſt Circle of it.

Wherefore, to 2rc, the Convex-Surface of the Cylinder, add rc equal to the Area of both its Baſes, you will have 3rc; which ſhews you, that the Surface of the Cylinder (including its Baſes) is to the Surface of the Sphere as 3 to 2; or that the Sphere is $\frac{2}{3}$ of the circumſcribing Cylinder, in Area as well as Solidity.

Or you may prove the Sphere to be $\frac{2}{3}$ of the Cylinder of the ſame Baſe and Altitude, by *Lemma* VI. aforegoing, thus:

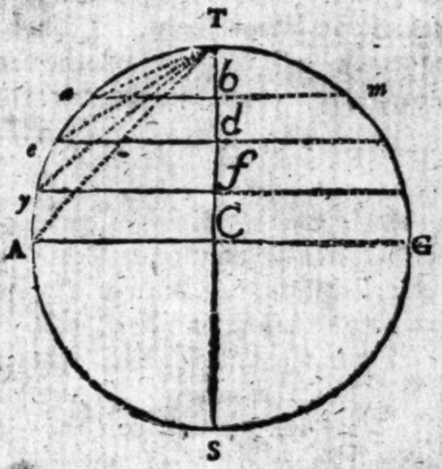


Let AGB repreſent the Hemiſphere, and AIKB half the Cylinder, then, if the Semidiameter GH be divided into ſix equal Parts, and Lines drawn parallel to AB, the Diameter, the Squares of the Semichords *ab*, *cd*, *ef*. &c. will be a Series of Numbers, whoſe greateſt Term AH is a ſquare Number, the others differing by odd Numbers; that is, AH is 36, *kl* 35, *gh* 32, *ef* 27, *cd* 20, *ab* 11: But an infinite Series of ſuch Numbers are in Proportion to an infinite Number of Terms equal to the greateſt, as 2 to 3. And becauſe the Hemiſphere is compoſed of an infinite Number of Circles, whoſe Diameters are the Chords of the Semicircle, and the half Cylinder is compoſed of an infinite Number of Circles, whoſe Diameters are all equal to the Diameter of the Semicircles A B; therefore the Hemiſphere is in Proportion to the half Cylinder as 2 to 3; and conſequently the whole Sphere bears the ſame Proportion to the whole Cylinder.

That the Superficies of every Sphere (or Globe) is equal to four times the Area of its greateſt Circle, is thus proved:

The Solidity of the Sphere is conſtituted of an infinite Number of parallel Circles, as is aforeſaid; conſequently the Superficies of the Sphere will be compoſed of the Peripheries of thoſe Circles which conſtitute its Solidity.

NOTE, In the following Demonſtrations, ⊙ ſignifies any Circle in general; and if any two Letters be joined to it, thus, ⊙AB, *&c.* then it denotes the Area of ſuch a Circle as thoſe two Letters repreſent the Radius of.



Det $D=TS$, the Axis of any Sphere, then, according to the Property of a Circle, it

will

will be	1	$D-Tb\times Tb=\square ab$;
that is,	2	$D\times Tb-\square Tb=\square ab$;
therefore	3	$D\times Tb=\square aT$.
For	4	$\square ab+\square Tb=\square aT$, (*Euc.* 1. 47.)
And	5	$D\times Td=\square eT$,
	6	$D\times Tf=\square yT$.

Hence it is evident, that the Series $\square aT$, $\square eT$, $\square yT$, *&c.* are in the ſame Ratio with T*b*, T*d*, T*f*, *&c. viz.* are in arithmetical Progreſſion: Whence it follows, that the $\odot aT=$ to the Sum of all the Circle's Peripheries between T and *b*.

And $\odot eT=$ the Sum of all the Circle's Peripheries between T and *d*, *&c.*

Conſequently, that the $\odot AT=$ the Sum of all the Circle's Peripheries, included between T and C; that is, $\odot AT=$ the Superficies of the Hemiſphere.

And becauſe $\square AC+\square TC=\square AT$, and $\square AC$ is equal to $\square TC$; therefore $\odot AT=2\odot AC$, is the Superficies of the Hemiſphere.

Conſequently, $4\odot AC$ will be the Superficies of the whole Sphere. Which was, *&c.*

SCHOLIUM.

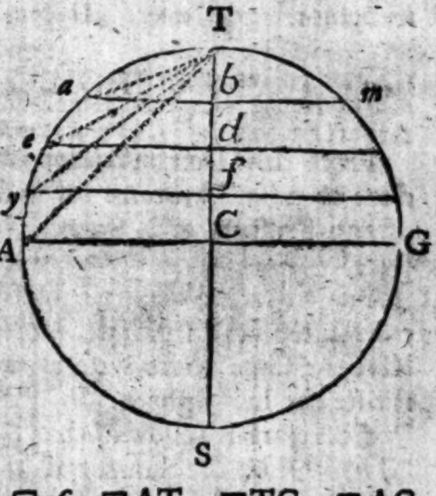
From the Method here uſed in proving the whole Superficies, it will be eaſy to find the Curve-Superficies of any Fruſtum, or Part of a Sphere, that is cut off by a right Line, or Plane, *viz.* ſuch as the Fruſtum *a*T*m* in the laſt Scheme, whoſe Curve Superficies is $\odot aT$, as above. Therefore (becauſe $\square ab+\square Tb=\square aT$) it will be $\odot ab+\odot Tb=$ the Curve-Superficies of that Fruſtum.

But if the Axis TS and Height T*b* of the Fruſtum are given, then it will be $TS\times Tb=\square aT$, as in the third Step above, which gives the Proportion or Theorem following, *viz.*

As the Axis of the Sphere : to the whole Superficies of the Sphere : : ſo is the Height of any Fruſtum : to its Curve-Superficies.

To which if there be added the Area of the Fruſtum's Baſe, the Sum will be the whole Superficies of the Fruſtum.

That the Solidity of every Sphere is two Thirds of its circumſcribing Cylinder, *may be thus proved.*



According to the Work above, it appears, that $\odot ab$, $\odot ed$, $\odot yf$, *&c*, do conſtitute the Solidity of the Sphere; and that $\square aT$, eT, yT, *&c.* are a Series of Terms in arithmetical Progreſſion, $\square AT$ being the greateſt Term, and TC the Number of Terms; therefore $\odot AT \times \frac{1}{2} TC =$ the Sum of all the Series, by *Lemma* II.

And becauſe $\square aT - \square Tb = \square ab$. $\square eT - \square Td = \square ed$. $\square yT - \square Tf = \square yf$. $\square AT - \square TC = \square AC$, *&c.* wherein $\square Tb$, $\square Td$, $\square Tf$, *&c.* are a Series of Squares, whoſe Roots Tb, Td, Tf, are in arithmetical Progreſſion; $\square TC$ being the greateſt Term, and TC the Number of Terms; therefore $\odot TC \times \frac{1}{3} TC =$ the Sum of all that Series, by *Lemma* III.

Conſequently, $\odot AT \times \frac{1}{2} TC : - \odot TC \times \frac{1}{3} TC =$ the Sum of all the Series $\odot ab$, $\odot ed$, $\odot yf$, *&c.* which conſtitute the Solidity of the half Sphere ATG. Put

$D=2TC$, the Axis of the Sphere, then $\frac{1}{4}D=\frac{1}{2}TC$, and $\frac{1}{6}D=\frac{1}{3}TC$. And becauſe $\square AT=2\boxdot TC$, therefore $\odot AT=2\odot TC=1.5708\,DD$; and $1.5708\,DD\times\frac{1}{4}D=0.3927\,DDD$.

Again, $\odot TC\times\frac{1}{3}TC=0.7854\,DD\times\frac{1}{6}D=.1309\,DDD$, then $0.3927\,DDD-0.1309\,DDD$, $=0.2618\,DDD$, the Solidity of the half Sphere.

Conſequently, $0.2618\,DDD\times 2=.5236\,DDD$, will be the ſolid Content of the whole Sphere, which is equal to $\frac{2}{3}$ of the Cylinder; the Diameter of whoſe Baſe, and alſo its Height, is $=D$.

For $0.7854\,DDD=$ the Solidity of the Cylinder, by Sect. V. But $\frac{2}{3}$ of $0.7854\,DDD=0.5236\,DDD$, as before.

SCHOLIUM.

From this Demonſtration it will be eaſy to deduce or raiſe Theorems for finding the ſolid Content of any Fruſtum of a Sphere, as Tm in the laſt Figure.

For we there ſuppoſe the Fruſtum aTm to be conſtituted of an infinite Series of Circles, which have the ſame Ratio with all thoſe Circles that conſtitute the half Sphere.

Therefore it follows, that $\odot aT\times\frac{1}{2}Tb:-\odot bT\times\frac{1}{3}Tb$, will be the Sum of all the Circles intercepted between T and b; conſequently it will be the Solidity of that Fruſtum.

And becauſe $\square ab+\square Tb=aT$, therefore $\odot ab+\odot Tb\times\frac{1}{2}Tb:-\odot Tb\times\frac{1}{3}Tb=$ the Solidity. Let $c=ab$, half the Diameter of the Fruſtum's Baſe; $h=Tb$ its Height, and $S=$ the Solidity of the Fruſtum. Then $\odot ab=3.1416cc$, and $\odot Tb=3.1416hh$; conſequently, $\frac{3.1416cch+3.1416hhh}{2}-$

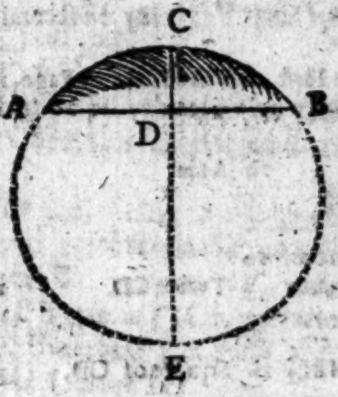
3.1416

$\frac{3.1416hhh}{3} = S$; which being reduced, will become $3cch + hhh \times 0.5236 = S$: Which is one Theorem for finding the Solidity of the Fruſtum, and may be expreſſed in Words thus:

If to three times the Square of the Semidiameter of the Fruſtum's Baſe you add the Square of the Height of the Fruſtum, and multiply the Sum by the Height of the Fruſtum, and that Product multiplied by .5236, the Product will be the ſolid Content.

But if the Axis of the Sphere, and the Height of the Fruſtum be given, then put $D =$ the Axis, $h =$ the Height of the Fruſtum, and c as before, it will be $D - h \times h = cc$, *viz.* $Dh - hh = cc$. Then will $3Dhh - 2hhh = 3cch + hhh$; conſequently $3Dhh - 2hhh \times 0.5236 = S$, the Fruſtum's Solidity; which is another Theorem for finding the Solidity of the Fruſtum, and may be expreſſed in Words thus:

From three times the Axis ſubtract twice the Height of the Fruſtum, and multiply the Remainder by the Square of the Height, and that Product multiply by .5236, this laſt Product will be the Solidity of the Fruſtum.



Example. Let ABCD be the Fruſtum of a Sphere; ſuppoſe AB (the Diameter of the Fruſtum's Baſe) be 16 Inches, and CD (the Height) 4 Inches; the Solidity is required.

By the firſt Rule.

8
8
64 Square of the Semidiameter AD.
3
192
16 Add the Square of CD.
208
4 Multiply by CD.
832

.5236
832
10472
15708
41888

The Solidity 435.6352

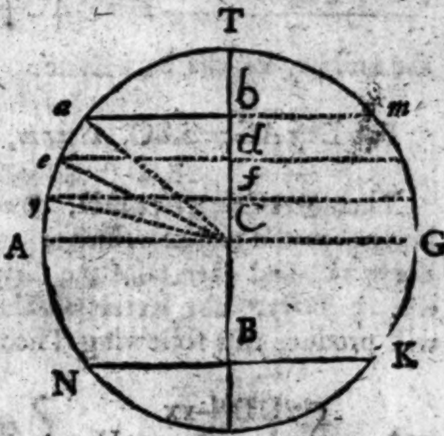
By the ſecond Rule, thus.

Firſt, by the Rule in Page 109, you will find the Axis of the whole Globe to be 29 Inches.

20 Axis
3
From 60
Subtr. 8 Twice CD.
Rem. 52
Mult. 16 Square of CD.
312
52
Prod. 832

.5236
832
10472
15708
41888
435.6352 } The Solid Content, the ſame as before.

And if it be required to find the middle Part *am* NK, uſually called the middle Zone of a Sphere;



Then becauſe it is ſuppoſed that $am = NK$, or (which is all one) that $bC = CB$, therefore it is plain, that if twice the Segment aTm be taken from the whole Sphere, there will remain the middle Zone $amNK$.

But becauſe that Work is a little troubleſome, I will here ſhew how to raiſe a Theorem for the doing it.

Firſt, becauſe $AC = yC = eC = aC = TC$, therefore it will be $\square AC - \square Cf = \square yf$, $\square AC - \square Cd = \square ed$, $\square AC - \square Cb = \square ab$, &c.

Here, becauſe $\square AC$, $\square AC$, $\square AC$, *&c.* are a Series of Equals, and Cb the Number of all the Terms; therefore $\square AC \times Cb =$ the Sum of all that Series, *per Lemma* I.

And $\square Cf$, $\square Cd$, $\square Cb$, &c, being a Series of Squares, whoſe Roots are in arithmetical Progreſſion, beginning at the Center C, *viz.* o. Cf, Cd, Cb, &c. wherein the greateſt Term is $\square Cb$, and the Number of Terms is Cb; therefore $\square Cb \times \frac{1}{3} Cb =$ the Sum of all the Series, *per Lemma* III.

Conſequently, the $\odot AC \times Cb : - \odot Cb \times \frac{1}{3} Cb =$ the Sum of all the Series $\odot yf$, $\odot ed$, $\odot ab$, &c. which do conſtitute the Solidity of the Half Zone $amAG$.

And becauſe $\square AC - \square CB = \square ab$, therefore $\odot AC - \odot ab = \odot Cb$. Conſequently $\odot AC \times Cb : - \frac{\odot AC + \odot ab : \times Cb}{3} = 2\odot AC + \odot ab : \times \frac{1}{3} Cb$ will be the Solidity of the half Zone.

Put $D = AG = 2AC$, $x = am$, and $H = bB = 2Cb$.

Then $\odot AC = .7854DD$, $\odot ab = .7854xx$. And if we turn the common Factor .7854 into a Diviſor, 1.27323, and then take the Triple of that Diviſor, *viz.* 3.8197, the Reſult of the precedent Work will produce this following Theorem.

$$\text{Theor.}\left\{\frac{2DD + xx}{3.8197} : \times H = \right\}\text{the middle Zone } amNK.$$

Which in Words is thus : To twice the Square of the Axis AG, add the Square of the Diameter of the Fruſtum's Baſe, (*am*) and divide the Sum by 3.8197; then multiply the Quotient by the Height or Thickneſs of the middle Zone, and the Product will be the Solidity of the middle Zone requir'd.

This is ſo plain and eaſy it needs no *Example*.

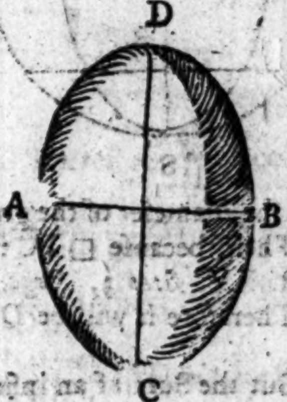
§ XII.

§ XII. *Of a* SPHEROID.

A Spheroid is a Solid reſembling an Egg. To find the ſolid Content thereof, this is

The RULE.

Multiply the Square of the Diameter of the greateſt Circle by the Length, and that Product multiply again by .5236, this laſt Product will be the Solidity of the Spheroid.



Let AB, the Diameter of the greateſt Circle, be 33 Inches, and CD, the Length, 55 Inches; the Solidity is required.

33
33

99
99

1089
55

5445
5445

59895

59895
.5236

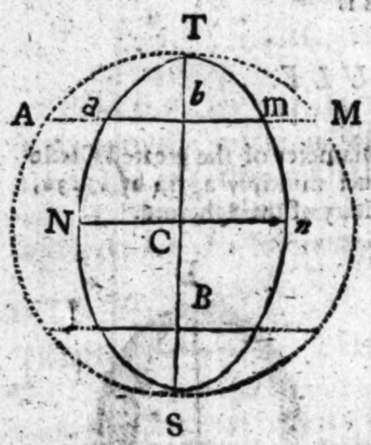
359370
179685
119790
299475

31361.0220 the Solidity

DEMON-

DEMONSTRATION.

Every Spheroid is equal to $\frac{2}{3}$ of a Cylinder, whoſe Baſe is equal to the greateſt Circle of the Spheroid, and its Height equal to the Length of the Spheroid.



Suppoſe the Figure NT*n*SN in the annex'd Scheme, to repreſent a Spheroid, formed by the Rotation of the Semi Ellipſis TNS, about its tranſverſe Axis TS.

Let D = TS, the Length of the Spheroid, and the Axis of its circumſcribing Sphere; and d = N*n* the Diameter of the greateſt Circle of the Spheroid.

Then, becauſe □TC : □NC : : □A*b* : □*ab*, by Sect. XV. *Step* 3, *Page* 119.

Therefore it will be DD : dd : : □A*b* : □*ab*.

But the Sum of an infinite Series of ſuch Circles as ⊙*ab* (whoſe Diameters are Chords) do conſtitute the Solidity of the Sphere (by Sect. XI.)

And the Sum of an infinite Series of ſuch Circles as ⊙*ab*. (*viz.* whoſe Diameters are Ordinates of the Ellipſis) do conſtitute the Solidity of the Spheroid.

Therefore DD : dd : : 0.5236DDD : 0.5236Ddd = the Solidity of the Spheroid. (*Eucl.* 5. 12.)

But 0.5236Ddd = $\frac{2}{3}$ of the Cylinder, whoſe Diameter is = d, and Height = D. (by Sect. V.)

Now, from this Proportion between the Sphere and its inſcribed Spheroid, it will be very eaſy to deduce Theorems for finding the ſolid Content either of the Fruſtum or middle Zone of any Spheroid, having the ſame Height with that of the Sphere; for,

As the Solidity of the whole Sphere : is to the Solidity of the whole Spheroid :: ſo is any Part of the Sphere, to the like Part of the Spheroid.

As for Inſtance, ſuppoſe it was required to find the middle Zone of any Spheroid.

Let $D=TS$ and $d=Nn$, as above, and $H=bB$. $x=AM$, and $c=am$.

Then $\left\{\frac{2DD+xx}{3.8197}\right.\times H=$ the middle Zone of the Sphere

And $0.5236DDD : 0.5236ddD :: \frac{2DD+xx}{3.8197}\times H :$ $\frac{2ddH}{3.8197}+\frac{xxddH}{3.8197DD}=$ the middle Zone of the Spheroid.

Again, $DD : dd :: xx : cc$. Therefore $\frac{xxdd}{DD}=cc$.

Conſequently, $\frac{xxdd}{DD}\times\frac{H}{3.8197}=\frac{cc}{3.8197}\times H$. Which being taken inſtead of $\frac{xxddH}{3.8197DD}$, there will ariſe This following Theorem, $\left\{\frac{2dd+cc}{3.8197}\right.\times H=$ the middle Zone of the Spheroid.

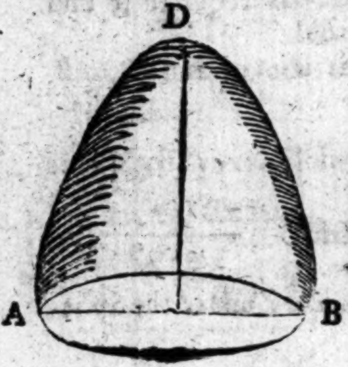
NOTE, That $3.8197=1.2732\times3$. *See p.* 101.

§ XIII. *Of a Parabolick* CONOID.

A Parabolick Conoid is ſomething like an half Spheroid, having its Sides ſomewhat ſtraighter. It is generated by ſuppoſing a Semi-Parabola turned about its Axis. To find the ſolid Content thereof, this is

The RULE.

Multiply the Square of the Diameter of its Baſe by .7854, and multiply that Product by half the Height, that laſt Product ſhall be the ſolid Content.



Let ABCD be a Parabolick Conoid, the Diameter of whoſe Baſe is 36 Inches, and its Height CD 33 Inches; the Solidity is required.

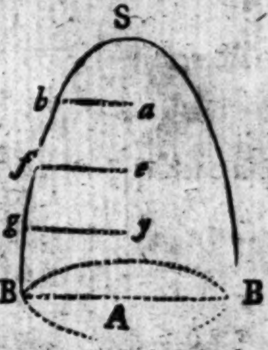
```
   36        .7854       1017.8784
   36         1296              33
 ----       ------      ----------
  216        47124        30536352
 108        70686        30536352
 ----      15708        ----------
 1296      7854         2)33589.9872
          ---------     ------------
          1017.8784       16794.9936

1728)16794.99|36(9.71 Feet, the Content.
     15552
     -----
      12429
      12096
      -----
        3339
        1728
        ----
        1611
```

DEMONSTRATION.

The Parabolick Conoid is conſtituted of an infinite Number of Circles, whoſe Diameters are the Ordinates of the Parabola. Now, according to the Property of every Parabola, it will be SA : AB :: AB : $\frac{\square AB}{SA} = L$, the *Latus Rectum.*

Then



$$\text{Then}\left\{\begin{array}{l} Sa\times L=\square ba,\\ Se\times L=\square fe,\\ Sy\times L=\square gy,\ \&c.\end{array}\right.$$

Here $Sa\times L$, $Se\times L$, $Sy\times L$, *&c.* are a Series of Terms in arithmetical Progreſſion.

Therefore $\square ba$, $\square fe$, $\square gy$, &c. are alſo a Series of Terms in the ſame Progreſſion, beginning at the Point S, wherein $\square AB$ is the greateſt Term, and SA the Number of all the Terms. Therefore $\square AB\times\frac{1}{2}SA$ = the Sum of all the Series. (by *Lemma* II.)

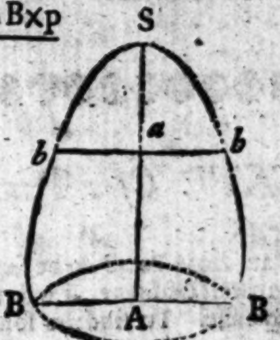
Conſequently, $\odot AB\times\frac{1}{2}SA$=the Sum of all the Series of $\odot ba$, $\odot fe$, $\odot gy$, &c. which do conſtitute the Solidity of the Conoid.

Put $D=2AB$, and $H=SA$.

Then $.7854DD\times\frac{1}{2}H=.3927DDH$ will be the ſolid Content of the Conoid, which is juſt half the Cylinder whoſe Baſe is $=D$, and Height $=H$.

This being rightly underſtood, it will be eaſy to raiſe a Theorem for finding the lower Fruſtum of any Parabolick Conoid.

For ſuppoſing $h=aA$, the Height of the Fruſtum, and $p=Sa$, the Height of the Part bSb cut off; then $h+p=SA$, the Height of the whole Conoid.



Conſequently $\frac{\odot AB\times h+\odot AB\times p}{2}$ =the Solidity of the whole Conoid.

And $\frac{\odot ba\times p}{2}$=the Solidity of the Part cut off.

Therefore

Therefore	1	$\frac{\odot AB\times h+\odot AB\times p-\odot ba\times p}{2}$, the Solidity of the Fruſtum.
But	2	$h+p:\square AB::p:\square ba$.
Conſequently	3	$h+p:\odot AB::p:\odot ba$.
$3\because$	4	$\odot AB\times p=\odot ba\times h+\odot ba\times p$.
$4-\odot ba\times p$	5	$\odot AB\times p:-\odot ba\times p=\odot ba\times h$.
$\overline{1\times 2}$	6	$\odot AB\times h:+\odot AB\times p:-\odot ba\times p=2F$.
$6-5$	7	$\odot AB\times h=2F:-\odot ba\times h$.
$7+\odot ba\times h$	8	$\odot AB\times h:+\odot ba\times h=2F$.
$\overline{8\div 2}$	9	$\frac{\odot AB+\odot ba}{2}:\times h=F$, the Fruſtum's Solidity.

Let $D=2AB$, as before, and $d=2ba$, the Diameter of the Part cut off; then we ſhall have this following Theorem.

$0.3927DD+0.3927dd:\times h=$ the Solidity of the Fruſtum required. Which in Words is thus:

Multiply the Sum of the Squares of the greater and leſſer Diameters by .3927, and that Product by the Height of the Fruſtum, the laſt Product ſhall be the ſolid Content.

§ XIV. *Of a Parabolick* SPINDLE.

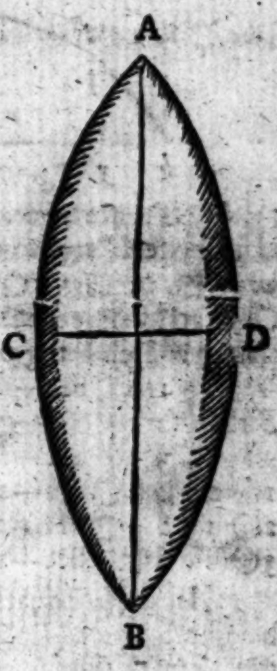
IF an acute Parabola be ſuppoſed to be moved about its greateſt Ordinate, it will form a Solid called a Parabolick Spindle. To find the ſolid Content, this is

The RULE.

Multiply the Square of the Diameter of its greateſt Circle by .41888 (being $\frac{8}{15}$ of .7854) and that Product by its Length, that laſt Product is the ſolid Content.

Let ABCD be a Parabolick Spindle, whoſe greateſt Diameter CD is 36 Inches, and its Length AB 99 Inches; the Solidity is required.

```
 36=CD           .41888
 36                1296
 ----           -------
 216             251328
108             376992
 ----           83776
1296 Square    41888
               ---------
               542.86848
                      99
               ---------
               488581632
              488581632
         -----------------
     1728)53743.97952(31,10184
              ......
         -----
         1903
          1759
            3179
            14515
              6912
            -------
              ....
```



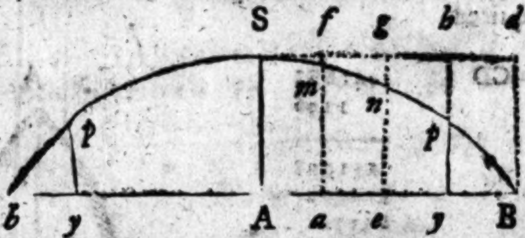
The ſolid Content is 31.1084 Feet.

DEMON-

DEMONSTRATION.

A Parabolick Spindle is conſtituted of an infinite Series of Circles, whoſe Diameters are all parallel to the Axis of the Parabola, as $\odot ma$, $\odot ne$, $\odot py$, *&c.*

Let us ſuppoſe the Line S*d* parallel to AB, *&c.* then it hath already been proved, that the Lines *fm*,



gn, *bp*, *&c.* are a Series of Squares whoſe Roots are in arithmetical Progreſſion; conſequently their Squares *viz.* $\square fm$, $\square gn$, $\square bp$, *&c.* will be a Series of Biquadrats, whoſe Roots will be in arithmetical Progreſſion: Which being premiſed, we may proceed thus:

First, | 1 | $SA - fm = ma.$
| 2 | $SA - gn = ne.$
| 3 | $SA - bp = py.$

$1 \circledcirc 2$ | 4 | $\square SA - 2SA \times fm + \square fm = \square ma.$
$2 \circledcirc 2$ | 5 | $\square SA - 2SA \times gn + \square gn = \square ne.$
$3 \circledcirc 2$ | 6 | $\square SA - 2SA \times bp + \square bp = \square py$, &c.

1. In theſe Equations, the $\square SA$, $\square SA$, $\square SA$, being a Series of Equals, and AB the Number of all the Terms; therefore it will be $\square SA \times AB =$ the Sum of all the Series, by *Lemma* I.

2. Becauſe *fm*, *gn*, *bp*, &c. are a Series of Squares wherein SA is the greateſt Term, and AB the Number of all the Terms;

therefore $\frac{2SA \times SA \times AB}{3} = \frac{2\square SA \times AB}{3}$ will be the Sum of all the Series, by *Lemma* III.

3. And the $\square fm$, $\square gn$, $\square bp$, &c. will be a Series of Terms in the Ratio of Biſquadrats, as above, $\square SA$ being the greateſt Term, and AB the Number of all the Terms; therefore it will be $\frac{\square SA \times AB}{5}$ = the Sum of all the Series, by *Lemma* V.

Whence it follows, that $\square SA \times AB - \frac{2\square SA \times AB}{3} + \frac{\square SA \times AB}{5}$ = the Sum of all the Series of $\square ma$, $\square ne$, $\square py$, &c.

That is, $\frac{8\square SA \times AB}{15}$ = the Sum of all the Series, $\square ma$, $\square ne$, $\square py$, &c. Conſequently, $\frac{8\odot SA \times AB}{15}$ = the Sum of all the Series of Circles, $\odot ma$, $\odot ne$, $\odot py$, &c. which conſtitute the Solidity of half the Spindle, *viz.* of SAB.

Therefore putting D = 2SA, and H = 2AB, it will be 0.41888 DDH = the Solidity of the whole Parabolick Spindle *b*SB, being $\frac{8}{15}$ of 0.7854 DDH, the Solidity of its circumſcribing Cylinder.

From hence we may alſo raiſe a Theorem for finding the Fruſtum SA*py*, of the laſt Figure. For $\odot SA$ being the greateſt Term, $\odot py$ the leaſt Term, and A*y* the Number of all the Terms or Circles included between A and *y*.

Therefore,

There fore, 1 $\left\{ \square SA-\frac{2SA\times bp}{3}+\frac{\square bp}{5}:\times Ay=z\right.$ the Sum of all the Series, $\square SA$, $\square ma$, $\square ne$, $\square py$.

$1\times\overline{3}$ 2 $3\square SA-3SA\times bp+\frac{3\square bp}{5}:\times Ay=3z$.

$2\div Ay$ 3 $3\square SA-2SA\times bp+\frac{3\square bp}{5}=\frac{3z}{Ay}$

But 4 $\square SA-2SA\times bp=\square py-\square bp$, per Step 6.

$3-4$ 5 $2\square SA+\frac{3\square bp}{5}=\frac{3z}{Ay}-\square py+\square bp$.

$5\pm$ &c. $\overline{6}$ $2\square SA+\square py-\frac{2}{5}\square bp=\frac{3z}{Ay}$

Conſeq. 7 $2\odot SA+\odot py-\frac{2}{5}\odot bp:\times\frac{1}{3}Ay=z$, the Sum of all the Series of $\odot SA$, $\odot ma$, $\odot ne$, $\odot py$, which do conſtitute the Solidity of the Fruſtum SA*py*. Therefore putting $D=2SA$, as before, $C=2py$, $x=2bp$, and $H=Ay$, it will be $1.5708\,DD+.7854\,CC-.31416xx:\times\frac{1}{3}H=$ the Fruſtum SA*py*. And if we make $L=2H$, then $1.5708DD+.7854\,CC-.31416xx:\times\frac{1}{3}L=$ the Double of that Fruſtum, being the middle Zone: Which in Words is thus.

Multiply the Square of the greateſt Diameter by 1.5708, and multiply the Square of the leſſer Diameter by .7854. and multiply the Square of the Difference of the Diameters by .31416; from the Sum of the two former Products ſubtract the latter Product, and multiply the Remainder by one third Part of the Length, and that Product will be the Solidity of the middle Zone required.

CHAP.

CHAP. III.

Of the Meaſuring of the Works of the ſeveral Artificers relating to Building; and what Methods and Cuſtoms are obſerved therein.

§ I. *Of* CARPENTERS WORK.

THE Carpenters Works which are meaſurable, are, Flooring, Partitioning and Roofing; all which are meaſured by the Square of 10 Feet long and 10 Feet broad; ſo that one Square contains 100 ſquare Feet.

1. *Of* Flooring.

If a Floor be 57 Feet 3 Inches long, and 28 Feet 6 Inches broad, how many Squares of Flooring are there in that Room?

Multiply 57 Feet 3 Inches by 28 Feet 6 Inches, and the Product is 1631 Feet, *&c.* which divide by 100; (this is done by cutting off from the Product two Figures towards the Right Hand with a Daſh of the Pen) and the remaining Figures are the Quotient, and the Figures cut off are Feet. Thus, 1631 divided by 100, by cutting off 31 from the Right Hand thereof, the Quotient is 16 Squares, and 31 cut off, is 31 Feet.

See the Work, both by Decimals, and alſo by Feet and Inches.

	F. I.
57.25	57 3
28.5	28 6
28625	456
45800	114
11450	28 7 6
	7 0 0
16|31.625	
	16|31 7 6

Facit, 16 Squares and 31 Feet.

NOTE, That .5 is the Decimal for Half of any Thing, .25 is the Decimal for a Quarter, and .125 is the Decimal for half a Quarter; ſo, in the laſt *Example*, 25 is the Decimal of 3 Inches, becauſe 3 Inches is a Quarter of a Foot; and 5 is the Decimal of 6 Inches, becauſe 6 Inches is half a Foot.

Example 2. Let a Floor be 53 Feet 6 Inches long, and 47 Feet 9 Inches broad, how many Squares are contained in that Floor?

	F. I.
47.75	53 6
53.5	47 9
23875	371
14325	212
23875	26 9
	13 4 6
25|54625	23 6
	25|54 7 6

Facit, 25 Squares and 54 Feet.

By Scale *and* Compaſſes.

In the firſt *Example*, extend the Compaſſes from 1 to 28.5, that Extent will reach from 57.25 to 16 Squares and near a third Part.

In the ſecond *Example*, extend the Compaſſes from 1 to 47.75, that Extent will reach from 53.5 to 25 Squares and above an half.

2. *Of* Partitioning.

Example 1. If a Partition between Rooms be in Length 82 Feet 6 Inches, and in Height 12 Feet 3 Inches, how many Squares are contained therein?

The Length and Breadth being multiplied together, the Product is 1010.625; which divide by 100, (as before is ſhewed) and the Anſwer is 10 Squares 10 Feet; the Inches or Parts in theſe Caſes are of no Value.

12.25	F.	I.	
82.5	82	6	
	12	3	
6125			
2450	990	0	
9800	20	7	6
10|10.625	10|10	7	6

Facit, 10 Squares 10 Feet.

Example 2. If a Partition between Rooms be in Length 91 Eect 9 Inches, and in Breadth 11 Feet 3 Inches, how many Squares are contained therein?

The Length and Breadth being multiplied together, the Product is 1032 Feet; which divided by 100, the Anſwer will be 10 Squares and 32 Feet.

91.75	F.	I.	
11.25	91	9	
	11	3	
45875			
18350	1009	3	
9175	22	11	3
9175			
10|32.1875	10|32	2	3

3 *Of* Roofing.

It is a Rule amongſt Workmen, that the Flat of any Houſe, and half the Flat thereof, taken within the Walls, is equal to the Meaſure of the Roof of the ſame Houſe; but this is when the Roof is true pitched. For if the Roof be more flat or ſteep than the true Pitch, it will meaſure to more or leſs accordingly.

Example 1. If a Houſe within the Walls be 44 Feet 6 Inches long, and 18 Feet 3 Inches broad, how many Squares of Roofing will cover that Houſe?

Multiply the Length and Breadth together, and the Product is 812 Feet, the Flat; the Half thereof is 406 Feet, which added to the Flat, the Sum is 1218 Feet; which divided by 100, the Anſwer is 12 Squares and 18 Feet.

18.25
44.5
9125
7300
7300
Flat. 812.125
Half. 406
12|18

F. I.
44 6
18 3
352
44
11 1 6
9 0 0
The Flat. 812 1 6
The Half. 406
Sum. 12'18

Facit, 12 Squares 18 Feet.

By Scale *and* Compaſſes.

In the firſt *Example* of Partitioning, extend the Compaſſes from 1 to 12.25; that Extent will reach from 82.5 to 10 Squares and one Tenth.

In the ſecond *Example*, extend the Compaſſes from 1 to to 11.25, that Extent will reach from 91.75 to 10 Squares, and a little leſs than a third Part.

In the *Example* of Roofing, extend the Compaſſes from 1 to 18,25, that Extent will reach from 44.5 to 812, the Flat; to which add the Half thereof, and the Sum is 12.18, which is 12 Squares 18 Feet, as above.

There are other Works about a Building done by the Carpenter, that are meaſured by the Foot, running Meaſure, that is, by the Number of Feet in Length only; as Cornices, Doors and Caſes, Window Frames, Guttering, Lintels, Sommers, Skirt boards, *&c.*

NOTE 1. In meaſuring of Flooring, after you have meaſured the whole Floor, you muſt deduct out of it the Well holes for the Stairs and Chimneys; and in Partitioning, for the Doors, Windows, *&c.* except (by Agreement) they are to be included.

NOTE 2 In meaſuring of Roofing, ſeldom any Deductions are made for the Holes for the Chimney Shafts, the Vacancies for Lutheren-Lights and Sky-Lights; for they are more Trouble to the Workman, than the Stuff which would cover them is worth.

§ II. *Of* BRICKLAYERS WORK.

THE principal is Tiling, Walling, and Chimney work.

1. *Of* Tiling.

Tiling is meaſured by the Square of 10 Feet, as Flooring, Partitioning and Roofing were in the Carpenter's Work; ſo that between the Roofing and Tiling the Difference will not be much, yet the Tiling will be the moſt; for the Bricklayers ſometimes will require to have double Meaſure for Hyps and Vallies. When Gutters are allowed double Meaſure, the Way is to meaſure the Length along the Ridge Tile, and by that Means the Meaſure of the Gutters becomes double; it is uſual alſo to allow double Meaſure at the Eaves, ſo much as the Projecture is over the Place, which is commonly about 18 or 20 Inches.

Example 1. There is a Roof covered with Tiles, whoſe Depth on both Sides (with the uſual Allowance at the Eaves) is 37 Feet 3 Inches, and the Length 45 Feet; I demand how many Squares of Tiling are contained therein?

F.	I.	
37	3	37.25
45	0	45
185		18625
148		14900
11	3	16|76.25
16|76	3	

Anſwer. 16 Squares 76 Feet.

Example 2. There is a Roof covered with Tiles, whoſe Depth on both Sides (with the Allowance at the Eaves) is 35 Feet 9 Inches and the Length 43 Feet 6 Inches; I demand how many Squares of Tiling are in the Roof?

F.	I.		
43	6		35.75
35	9		43.5
215			17875
129			10725
21	9		14300
10	10	6	
17	6		15|55.125
15|55	1	6	

Here the Length and Depth being multiplied together, the Product is 1555 Feet; which divided by 100, (as before is taught) the Anſwer is 15 Squares and 55 Feet.

By Scale *and* Compaſſes.

In the firſt *Example*, extend the Compaſſes from 1 to 37.25, that Extent will reach from 45 to 16 Squares, and a little above three Quarters of a Square.

In the ſecond *Example*, extend the Compaſſes from 1 to 35.75, that Extent will reach from 43.5 to 15 Squares and 55 Feet, that is, a little above a half Square.

2. *Of* Walling.

Bricklayers commonly meaſure their Work by the Rod-ſquare of 16 Feet and a half; ſo that one Rod in Length, and one in Breadth, contain 272.25 ſquare Feet; for 16.5 multiplied in itſelf, produces 272.25 ſquare Feet: But in ſome Places the Cuſtom is to allow 18 Feet to the Rod, that is, 324 ſquare Feet; and in ſome Places the uſual Way is to meaſure by the Rood of 21 Feet long and 3 Feet high, that is 63 ſquare Feet; and here they never regard the Thickneſs of the Wall, but the uſual Way is to moderate the Price according to the Thickneſs,

But in *Ireland* they meaſure by a Perch of 21 Feet in Length and 1 Foot in Height, without any regard to Thickneſs in the Meaſure.

When you meaſure a Piece of Brick-work, the firſt Thing is to enquire by which of thoſe Ways it muſt be meaſured; then having multiplied the Length and Breadth in Feet together, divide the Product by the proper Diviſor, either for Rods or Roods, and the Quotient is ſquare Rods accordingly.

But in *England*, commonly Brick Walls, that are meaſured by the Rod, are to be reduced to a Standard Thickneſs, *viz.* of a Brick and a half thick, (if it be not agreed on to the contrary) and to reduce a Wall to Standard Thickneſs, this is

The RULE.

Multiply the Number of ſuperficial Feet that are found to be contained in any Wall, by the Number of half Bricks which that Wall is in Thickneſs, one third Part of that Product ſhall be the Content thereof in Feet, reduced to the Standard Thickneſs of one Brick and a half.

Example 1. If a Wall be 72 Feet 6 Inches long, and 19 Feet 3 Inches high, and 5 Bricks and a half thick, how many Rods of Brick-work are contained therein when reduced to the Standard?

19.25 Height.
72.5 Length.

9625
3850
13475

1395.625
11

3)15351.875

272.25) 5117.291(18 Rods.

239479

68.06)21679(3 Quarters of a Rod.

12.61
Anſwer. 18 Rods, 3 Quarters, 12 Feet

F. I.
72 6
19 3

648
72
18 1 6
9 6 0

1395 7 6
11

3)15345

272)5115(18 Rods.

2395

68)219(3 Quarters of a Rod.

15

NOTE, That 68.06 is one fourth Part of 272.25.

NOTE alſo, That in reducing of Feet into Rods, they uſually reject the odd Parts, and divide only by 272, as is done in the ſecond Way of the laſt *Example*; ſo the Anſwer, by that ſecond Way, is 18 Rods, 3 Quarters, and 15 Feet, more by about 2 ½ Feet than by the firſt Way, where it is done decimally; a Thing very inſignificant.

Example 2. If a Wall be 245 Feet 9 Inches long, and 16 Feet 6 Inches high, and two Bricks and a half thick. I demand how many Rods of Brick-work are contained therein, when reduced to Standard Thickneſs?

245.75
16.5

122875
147450
24575

4054.875
5

3)20270

272)6756(24 Rods.

1316

68)228(3 Quarters of a Rod.

24

Anſwer. 24 Rods, 3 Quarters, 24 Feet.

F.	I.	
245	9	
16	6	
1470		
245		
122	10	6
12	0	0
4054	10	6 *Anſwer* in Feet.

Before I ſhew how to work the two laſt *Examples* by Scale and Compaſſes, I will ſhew how to find proper Diviſors to facilitate the Operation, becauſe it would be too intricate and tedious to perform by Scale and Compaſſes, according to the Rule above taught.

To find the proper Diviſors.

Divide 3 (the Number of Bricks in 1 and $\frac{1}{2}$) by the Number of half Bricks in the Thickneſs, the Quotient will be a Diviſor which will give the Anſwer in Feet. But if you would have a Diviſor to bring the Anſwer into Rods at once, then multiply 272.25 by the Diviſor found for Feet, and the Product will be a Diviſor which will give the Anſwer in Rods.

Example. Let it be required to find a Diviſor proper to reduce a Wall of three Bricks thick.

Divide 3 by 6, (the half Bricks in the Thickneſs) and the Quotient is 5, which is a Diviſor that will give the Anſwer in Feet. Then multiply 272.25 by .5, and the Product is 136.125, the Diviſor which will give the Anſwer in Rods; that is, as 136.125 is to the Length of the Wall, ſo is the Height to the Content in Rods. Or, as .5 is to the Length, ſo is the Height to the Content in Feet.

After the ſame Manner you may find Diviſors for any other Thickneſs, which you will find to be as expreſſed in the following little Table.

The

The Thickneſs of the Wall.	Diviſors for the Anſwer in Feet.	Diviſor for bringing the Anſwer in Rods.
1 Brick thick	1.5	408.375
1 and half Brick thick	1.	272.25
2 Bricks thick	.75	204.1875
2 and half Bricks thick	.6	163.35
3 Bricks thick	.5	136.125
3 and half Bricks thick	.4285	116.678
4 Bricks thick	.375	102.0937

Let the ſecond *Example*, aforegoing, be wrought by Scale and Compaſſes, where the Length is 245.75, the Height 16.5, and the Thickneſs is 2 and half Bricks.

Extend the Compaſſes from 163.35 (the tabular Number againſt 2 and half Bricks) to 245.75, that Extent will reach from 16.5 to 24 Rods and 8 Tenths.

Again, if the Length be 75 Feet 6 Inches, and the Height 18 Feet 9 Inches, at 3 and half Bricks thick, how many Rods are contained therein?

Extend the Compaſſes from 116.678 (the tabular Number) to 18.75, that Extent will reach from 75.5 to 12.13; that is, 12 Rod, and a little above half a Quarter.

In *Britain* it will be very proper and commodious, for ſuch as have frequent Occaſion to meaſure Brick-work, to have in the Line of Numbers little Braſs Center-pins at each of the Numbers in the third Column of the above little Table, with a Figure to denote the Thickneſs of the Wall.

If a Wall be 104 Feet 9 Inches long, and 17 Feet 3 Inches high, how many Rods are contained therein?

	F.	I.	
104.75	104	9	
17.25	17	3	
52375	728		
20950	104		
73325	26	2	3
10475	12	9	0
63)1806.9375(28	1806	11	3
126			
546			
504			
42			

Anſwer. 28 Rods, 42 Feet.

NOTE, That ſuch as dig Cellars do many times do them by the Floor, 18 Feet ſquare, and a Foot deep, being a Floor of Earth, that is, 324 ſolid Feet.

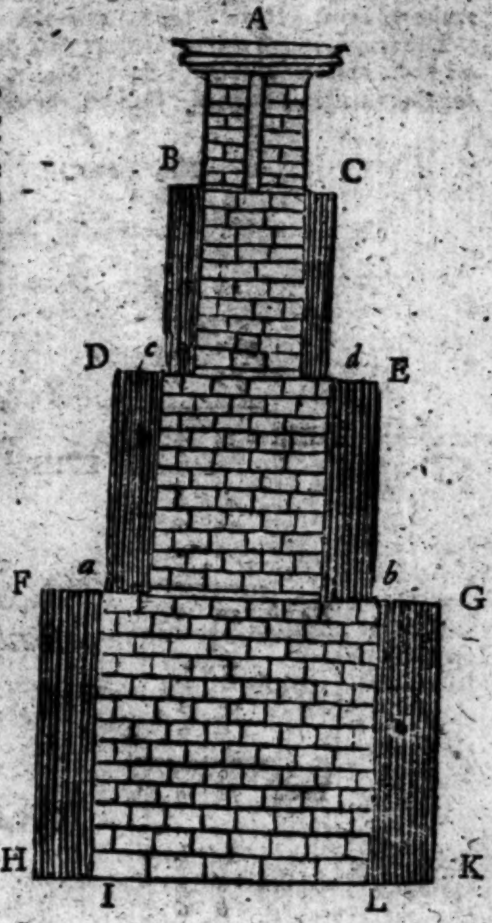
3. *Of* Chimnies.

If you are to meaſure a Chimney ſtanding alone by itſelf, without any Party-wall being adjoined, then girt it about for the Length, and the Height of the Story is the Breadth; the Thickneſs muſt be the ſame as the Jaums are of, provided that the Chimney be wrought upright from the Mantle-tree to the Cieling, not deducting any Thing for the Vacancy between the Floor (or Hearth) and the Mantle-tree, becauſe of the Gatherings of the Breaſt and Wings, to make room for the Hearth in the next Story.

If the Chimney back be a Party-wall, and the Wall be meaſured by itſelf, then you muſt meaſure the Depth of the two Jaums, and the Length of the Breaſt for a Length, and the Height of the Story the Breadth, at the ſame Thickneſs your Jaums were of.

When you meaſure Chimney-ſhafts, girt them with a Line round about the leaſt Place of them, for the Length, and the Height ſhall be your Breadth: And if they be four Inch Work, then you muſt ſet down your Thickneſs at one Brick-work; but if they be wrought 9 Inches thick, (as ſometimes they are, when they ſtand high and alone above the Roof) then you muſt account your Thickneſs one and half Brick, in conſideration of Wyths and Pargetting, and Trouble in Scaffolding.

It is cuſtomary, in moſt Places to allow double Meaſure for Chimnies.



Example. Suppoſe this Figure, ABCDEFGHK, to be a Chimney that hath a double Funnel towards the Top, and a double Shaft, and is to be meaſured according to double Meaſure.

Firſt, I begin with the Breaſt-Wall IL, and the two Angles LK and HI, which together are 18 Feet 9 Inches; then take the Height of the Square HF, 12 Feet 6 Inches, which multiply'd together, produce 234 Feet, 4 Inches, 6 Parts, for the Content of the Figure FGHK.

For the Square D*a*E*b*, the Length of the Breaſt Wall and two Angles, is 14 Feet 6 Inches, and the Height D*a* 9 Feet, which multiplied together make 130 Feet 6 Inches, for the Content of the Square D*a*E*b*.

Then the Height of the next Square 7 Feet, and the Length of the Breaſt-Wall and two Angles is 10 Feet 3 Inches, which multiplied together produceth 71 Feet 9 Inches, for the Content of the Square B*c*C*d*.

The Compaſs of the Chimney-Shafts is 13 Feet 9 Inches, and the Height 6 Feet 6 Inches, which multiplied together make 89 Feet, 4 Inches, 6 Parts, the Content of the Shafts.

The Depth of the middle Fetter, that parts the Funnels, is 12 Feet, and its Wideneſs 1 Foot 3 Inches, which multiplied together make 15 Feet, the Content thereof.

The Work.

	F.	I.			
	18	9			18.75
	12	6			12.5
	225	0			9375
	9	4	6		3750
					1875
FGHK	234	4	6	FGHK	234.375
	F.	I.			
	14	6			14.5
	9	0			9
DaEb	130	6		*DaEb*	130.5
	F.	I.			
	10	3			10.25
	7	0			7
BcCd	71	9		*BcCd*	71.75
	F.	I.			
	13	9			13.75
	6	6			6.5
	82	6			6875
	6	10	6		8250
The Shaft	89	4	6	The Shaft	89.375

	F.	I.
	1	3
	12	0
The Fetter	15	0

1.25
12
The Fetter 15.00

	F.	I.	P.
FGHK	234	4	6
DaEb	130	6	0
BcCd	71	9	0
The Shaft	89	4	6
The Fetter	15	0	0
The Sum	541	0	0
The Double	1082	0	0

272)1082(3 Rods.
68) 266(3 Quarters.
Rem. 62 Feet.

Having added the five Products together, and doubled the Sum, that double Sum is the Content of the Chimney in Feet, according to double or cuſtomary Meaſure; which Feet muſt be reduced to Rods, as was ſhewed before.

So the Feet in the foregoing *Example* being reduced to Rods, (the Thickneſs being ſuppoſed 1 and half Bricks) it makes 3 Rods, 3 Quarters, and 62 Feet, that is, 4 Rods wanting 6 Feet.

This is all the Meaſure that can be allowed, when the Chimney ſtands in a Gavel, or Side Wall; in which Caſe, the Back of the Chimney (here not meaſured) is accounted as Part of the Gavel; but if the Chimnies ſtand by themſelves as all Stacks of Chimnies in great Buildings do, which, in ſuch Caſe, is all Chimney-work, and therefore ought to be meaſured double on all Sides.

* * *

§ III. *Of* PLAISTERERS Work.

THE Plaiſterers Works are principally of two Kinds, namely, 1. Works lath'd or plaiſter'd, which they call Cieling. 2. Works render'd, which is of two Kinds, *viz.* upon Brick Walls, or between Quarters in the Partitions between Rooms; all which are meaſured by the Yard ſquare, or Square of 3 Feet, which is 9 Feet.

1. *Of* Cieling.

If a Cieling be 59 Feet 9 Inches long, and 24 Feet 6 Inches broad, how many Yards doth that Cieling contain?

Multiply 59 Feet 9 Inches by 24 Feet 6 Inches, and the Product is 1463 Feet, 10 Inches, 6 Parts; which divided by 9, the Quotient is 162 Yards, 5 Feet.

F. I.		59.75
59 9		24.5
24 6		
		29875
236		23900
118		11950
29 10 6		
18 0 6		9)1463.875
1463 10 6	*Anſwer.*	162 5

By Scale *and* Compaſſes.

Extend the Compaſſes from 9 to 59 Feet 9 Inches, that Extent will reach from 24 Feet 6 Inches to 162.5 Yards.

2. *Of* Rendering.

Example. If the Partitions between Rooms be 141 Feet 6 Inches about, and 11 Feet 3 Inches high, how many Yards are in thoſe Partitions?

Multiply 141 Feet 6 Inches by 11 Feet 3 Inches, and the Product is 1591 Feet, 10 Inches, 6 Parts; which divided by 9, gives 176 Yards 7 Feet, the Anſwer.

141 6
11 3

1556 6
35 4 6

9)1591 10 6

F. Y.
Anſw. 176 7

141.5
11.25

7075
2830
1415
1415

9)1591.875

176.87

Anſwer. 176.87 Yards.

Extend the Compaſſes from 9 to 141.5, that Extent will reach from 11.25 to 176.87 Yards.

NOTE 1. If there be any Doors, Windows, or the like, in your Partitioning, you muſt make Deductions for them.

NOTE 2. When you meaſure rendering upon Brick-Walls you are to make no Deductions; but when you meaſure Rendering between Quarters, you may very well deduct one fifth Part for the Quarters, Braces and Interſtices.

NOTE 3. That Whiting and Colouring are both meaſured by the Yard, as Cieling and Rendering were; and as in Rendering between Quarters you deduct one fifth Part, so in Whiting and Colouring you muſt add one fourth or one fifth Part, at leaſt.

§ IV. Of JOYNERS Work.

JOYNERS do meaſure their Work by the Yard ſquare; but in taking their Dimenſions they differ from ſome others; for they have a Cuſtom, and ſay, *We ought to meaſure where our Plane touches*; wherefore, in taking the Height of any Room, where there is a Cornice about, and ſwelling Pannels and Mouldings, they, with a String, begin at the Top, and girt over all the Mouldings, which will make the Room to meaſure much higher than it is: Then, for meaſuring about the Room, they only take it as it is upon the Floor.

Example 1. If a Room of Wainſcot (being girt downwards over the Mouldings) be 15 Feet 9 Inches high, and 126 Feet 3 Inches in compaſs, how many Yards doth that Room contain?

Multiply the Compaſs by the Height, and the Product is 1988 Feet, 5 Inches, 3 Parts; which divided by 9, gives 220 Yards and 8 Feet, the Anſwer.

F.	I.	
126	3	
15	9	
630		
126		
63	1	6
31	6	9
3	9	0
9)1988	5	3
Anſwer. 220	8	

126.25
15.75

63125
88375
63125
12625

9)1988.4375

220 8

Facit 220 Yards, 8 Feet.

Example 2. If a Room of Wainſcot be 16 Feet, 3 Inches high (being girt over the Mouldings) and the Compaſs of the Room 137 Feet 6 Inches, how many Yards are contained therein?

Multiply 137 Feet 6 Inches by 16 Feet 3 Inches, and the Product is 2234 Feet, 4 Inches, 6 Parts; which divided by 9, the Quotient is 248 Yards and 2 Feet.

```
   F.  I.
   137  6                    137.5
    16  3                    16,25
  ---------                 -------
   830                        6875
  137                        2750
     34  4  6               8250
  -----------              1375
 9)1234  4  6             --------
  -----------             9)2234.375
   248  2  0              ---------
                            248  2
```

Facit 248 Yards, 2 Feet.

By Scale *and* Compaſſes.

For the firſt *Example*, extend the Compaſſes from 9 to 126,25, that Extent will reach from 15.75 to 220.8 Yards.

For the ſecond *Example*, extend the Compaſſes from 9 to 137.5, that Extent will reach from 16.25 to 248 Yards, and about a Quarter.

In Joyners Work there is another Thing to be obſerved, that is, in the meaſuring of Doors, Window-Shutters, and all ſuch Work as is wrought on both Sides, they are paid for Work and half Work; ſo that in meaſuring all ſuch Work, you muſt firſt find the Content, as before, and take half that Content and add to it; ſo ſhall the Sum be the Content at Work and half.

Example. If the Window-Shutters about a Room be 69 Feet 9 Inches broad, and 6 Feet 3 Inches high, how many Yards are contained therein at Work and half?

Multiply 69 Feet 9 Inches by 6 Feet 3 Inches, and the Product is 435 Feet, 11 Inches, 3 Parts, the half whereof is 217 Feet, 11 Inches, 7 Parts; which added together, the Sum is 653 Feet, 10 Inches, 10 Parts; which divided by 9, the Quotient is 72 Yards 5 Feet, the Content at Work and half.

F. I.	69.75
69 9	6.25
6 3	
	34875
418 6	13950
17 5 3	41850
435 11 3	435.9375
217 11 7	217.9687
9)653 10 10	653.9062
72 5	

Facit 72 Yards 5 Feet.

By Scale *and* Compaſſes.

Extend the Compaſſes from *9* to *69.75*, that Extent will reach from *6.25* to 48.4 Yards; the Half whereof is 24.2, which added together make 72.6 Yards, the Content at Work and half.

NOTE, That you muſt make Deductions for all Window-Lights; but you muſt meaſure the Window-Boards, Sopheta-Boards and Cheeks by themſelves.

* * *

§ V. *Of* PAINTERS Work.

THE taking the Dimenſions of Painters Work is the ſame as that of Joyners, by girting over the Mouldings and Swelling Pannels, in taking the Height; and it is but Reaſon that they ſhould be paid for that on which their Time and Colour are both expended. The Dimenſions thus taken, the caſting up, and reducing Feet into Yards, is altogether the ſame as the Joyners Work; but the Painter never requires Work and half, but reckons his Work once, twice, or thrice coloured over. Only take Notice,

tice, that Window-Lights, Window-Bars, Caſements, and ſuch like Things, they do at ſo much *per* Piece.

Example. If a Room be painted, whoſe Height (being girt over the Mouldings) is 16 Feet 6 Inches, and the Compaſs of the Room 97 Feet 9 Inches, how many Yards are in that Room?

Multiply 97 Feet 9 Inches by 16 Feet 6 Inches, and the Product is 1612 Feet, 10 Inches, 6 Parts; which being divided by 9, the Quotient is 179 Yards and 1 Foot.

```
     F.  I.
     97  9                97.75
     16  6                 16.5
   ----------           ---------
    584                   48875
    98                   58650
    4810.6               9775
  ------------          ---------
 9)1612 10  6           1612.875
  ------------
    179  1
```

Facit 179 Yards, 1 Foot.

By Scale *and* Compaſſes.

Extend the Compaſſes from 9 to 16.5, that Extent will reach from 97.75 to 179.2 Yards.

* * *

§ VI. *Of* GLASIERS Work.

GLASIERS do meaſure their Work by the Foot ſquare; ſo that the Length and Breadth of a Pane of Glaſs in Feet, being multiplied into each other, produceth the Content.

NOTE, That Glaſiers do uſually take their Dimenſions to a Quarter of an Inch; and in multiplying Feet, Inches and Parts, the Inch is divided into 12 Parts, as the Foot is, and each Part ſubdivided into 12, *&c.*

Example 1. If a Pane of Glaſs be 4 Feet, 8 Inches and 3 Quarters long, and 1 Foot, 4 Inches, 1 Quarter broad, how many Feet of Glaſs in that Pane?

$$\text{The Decimal of}\left\{\begin{array}{l}8\text{ Inches }\frac{3}{4}\\4\text{ Inches }\frac{1}{4}\end{array}\right\}\text{ is }\left\{\begin{array}{l}.729\\.354\end{array}\right.$$

F. I. P.	
4 8 9	4.729
1 4 3	1.354
4 8 9	18916
1 6 11 0	23645
1 2 2 3	14187
	4729
6 4 10 2 3	6.403066

Anſwer. 6 Feet, 4 Inches.

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 1.354, that Extent will reach from 4.729 to 6.4 Feet, the Content.

Example 2. If there be eight Panes of Glaſs, each 4 Feet, 7 Inches, 3 Quarters long, and 1 Foot, 5 Inches, 1 Quarter broad, how many Feet of Glaſs is contained in the ſaid eight Panes?

$$\text{The Decimal of}\left\{\begin{array}{l}7\text{ Inches }\frac{3}{4}\\5\text{ Inches }\frac{1}{4}\end{array}\right\}\text{ is }\left\{\begin{array}{l}.646\\.437\end{array}\right.$$

F. I.

```
F. I. P.                    4.646
4  7  9                     1.437
1  5  3                  ————————
————————                    32522
4  7  9                    13938
1 11  2  9                18584
   1  1 11  3            4646
——————————————          —————————
6  8  1  8  3            6.676302
            8                   8
——————————————          —————————
53 5  1  6  0            53.410416
```

Facit 53 Feet, 5 Inches.

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 1.437, that Extent will reach from 4.646 to 6.676; then extend the Compaſſes from 1 to 8, that Extent will reach from 6.676 to 53.4, the Content.

Example 3. If there be 16 Panes of Glaſs, each 4 Feet, 5 Inches and a half long, and 1 Foot, 4 Inches, 5 Quarters broad, how many Feet of Glaſs are contained therein?

```
F. I. P.                    4.458
4  5  6                     1.395
1  4  9                  ————————
————————                    22290
4  5  6                    40122
1  5 10  9                13374
   2  4  1  6             4458
——————————————          —————————
6  2  8  1  6             6.218910
            4                   4
——————————————          —————————
24 10 8  6  0             24.87564
            4                   4
——————————————          —————————
99 6 10  0  0             99.50256
```

Facit 99 Feet, 6 Inches.

NOTE, That inſtead of multiplying by 16, I have multiplied by 4 twice, becauſe 4 times 4 is 16.

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 1.395, that Extent will reach from 4.458 to 6.219; then extend the Compaſſes from 1 to 16, that Extent will reach from 6.219 to 99.5 Feet, the Content.

NOTE, That when Windows have half Rounds at the Top, they meaſure them at the full Height, as if they were ſquare. Alſo round or oval Windows are meaſured at the full Length and Breadth of their Diameters. Likewiſe Crocket Windows in Stone-work are all meaſured by their full Squares; and there is Reaſon for ſo doing, for the Trouble in taking their Dimenſions to work by, the Waſte of Glaſs in working, and the Time expended in ſetting up, is far more than the Glaſs can be valued at.

◆ ◆ ◆

§ VII. *Of* MASONS Work.

MASONS do meaſure their Work ſometimes by the Foot ſolid, ſometimes by the Foot ſuperficial; and in ſome Places they meaſure their Walling by the Rood that is 21 Feet long and 3 Feet high, which is 63 ſquare Feet.

Examples of each are as follow.

Example 1. If a Wall be 97 Feet 5 Inches long, 18 Feet 3 Inches high, and 2 Feet 3 Inches thick, how many ſolid Feet are contained in that Wall?

F.	I.			
97	5			97.417
18	3			18.25
776				487085
97				194834
24	4	3		779336
6	0	0		97417
1	6	9		
1777	10	3		1777.86025
2	3			2.25
				888930125
3555	8	6		355572050
444	5	6	9	355572050
4000	2	0	9	4000,1855625

Multiply the Length, Height and Thickneſs together, and the laſt Product is 4000 Feet 2 Inches, the ſolid Feet contained in the Wall.

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 18.25, that Extent will reach from 97.417 to 1777.86; then extend from 1 to 1777.86, that Extent will reach from 2.25 to 4000,18, the ſolid Content.

Example 2. If a Wall be 107 Feet 9 Inches long, and 20 Feet 6 Inches high, how many Feet ſuperficial are contain'd therein?

F.	I.		
107	9		107.75
20	6		20.5
2155	0		53875
53	10	6	21550
2208	10	6	2208.875

Facit 2208 Feet 10 Inches.

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 107.75, that Extent will reach from 20.5 to 2208.875, the ſuperficial Feet.

Example 3. If a Wall be 112 Feet 3 Inches long, and 16 Feet 6 Inches high, how many Roods are contained therein?

F.	I.		
112	3		112.25
16	6		16.5
676	9		56125
112			67350
56	1	6	11225
1852	1	6	63)1852.125(29
			592

Facit 29 Roods 25 Feet. 25

By Scale *and* Compaſſes.

Extend the Compaſſes from 63 to 16.5, that Extent will reach from 112.25 to 29.4 Roods, the Content.

CHAP.

CHAP. IV.

The Meaſuring of BOARD *and* TIMBER.

§ I. *Of* BOARD-MEASURE.

TO meaſure a Board, is no other but to meaſure a long Square.

Example 1. If a Board be 16 Inches broad, and 13 Feet long, how many Feet is contained therein?

Multiply 16 by 13, and the Product is 208; which divided by 12, gives 17 Feet, and 4 remains, which is a third Part of a Foot.

Or thus: Multiply 156 (the Length in Inches) by 16, and the Product is 2496; which divided by 144, the Quotient is 17 Feet, and 48 remains, which is the third Part of 144; the ſame as before.

As 12 : 13 :: 16
13
48
16
12)208
17 $\frac{4}{12}$

Or, As 144 : 156 :: 16
16
936
156
144)2496(17 $\frac{48}{144}$
1056
48

By Scale *and* Compaſſes.

Extend the Compaſſes from 12 to 13, that Extent will reach from 16 to 17 $\frac{1}{3}$ Feet, the Content.

Or, Extend from 144 to 156, (the Length in Inches) that Extent will reach from 16 to 17 $\frac{1}{3}$ Feet, the Content.

Example 2. If a Board be 19 Inches broad, how many Inches in Length will make a Foot?

Divide 144 by 19, and the Quotient is 7.58 very near; and ſo many Inches in Length, if a Board be 19 Inches broad, will make a Foot.

Inch. Inch. Inch. Inch.
As 19 : 144 :: 1 : 758 *ferè.*

Extend the Compaſſes from 19 to 144, that Extent will reach from 1 to 7.58, that is, 7 Inches and ſomething more than a half. So if a Board be 19 Inches broad, if you take 7 Inches and a little more than a half with your Compaſſes from a Scale of Inches, and run that Extent along the Board, from End to End, you may find how many Feet that Board contains, or you may cut off from that Board any Number of Feet deſir'd.

For this Purpoſe there is a Line upon moſt ordinary Joint-Rules, with a little Table placed upon the End, of all ſuch Numbers as exceed the Length of the Rule, as in this little Table annexed.

I.	0	0	0	0	5	0	8½	6	Long
F.	12	6	4	3	2	2	1	1	Long
I.	1	2	3	4	5	6	7	8	Broad

Here you ſee, if the Breadth be one Inch, the Length muſt be 12 Feet; if 2 Inches, the Length is 6 Feet; if 5 Inches broad, the Length is about 2 Feet 5 Inches, *&c.*

The reſt of the Lengths are expreſſed in the Line, thus: If the Breadth be 9 Inches, you will find it againſt 16 Inches, counted from the other End of the Rule: if the Breadth be 11 Inches, then a little above 13 Inches will be the Length of a Foot, *&c.*

* * *

§ II. *Of* SQUAR'D TIMBER.

BY ſquar'd Timber is here meant all ſuch as have equal Baſes, and the Sides ſtrait and parallel. The Rules for meaſuring all ſuch Solids are ſhewed in Section II. of Chap 2. to which I refer you.

Example 1. If a Piece of Timber be 1 Foot 3 Inches (or 15 Inches) ſquare and 18 Feet long, how many ſolid Feet are contained therein?

```
         15
         15                 F.  I.
        ----
         75                 1   3
        15                  1   3
       -----              ----------
        225                 1   3
         18                     3   9
       -----              ----------
       1800                 1   6   9
        225                         6
       -----              ----------
 144) 4050(28.125           9   4   6
       -----                        3
       1170               ----------
        180                28   1   6
         360
          720
```

Anſwer. 28 Feet and half a Quarter.

Here, inſtead of multiplying by 18, (where I wrought by Feet and Inches) I multiplied by 6, and then by 3, becauſe 3 times 6 is 18.

Example 2. If a Piece of ſquar'd Timber be 2 Feet 9 Inches deep, and 1 Foot 7 Inches broad, and 16 Feet 9 Inches long, how many Feet of Timber are in that Piece?

Multiply the Depth, Breadth and Length together, and the Product will be the Content.

```
         33
         19
       ----            F.  I.
        297             2  9
         33             1  7
       ----            -------
        627             2  9
      16.75             1  7  3
      -----            -------
       3135             4  4  3
      4389             16  9
     3762              ---------
     627              69  8  0
     -----             3  3  2  3
144)10502.25(72.93    ----------
      -----           72 11  2  3
       422
      1342
        465
      -----
         33
```

Anſwer. 72 Feet 11 Inches, or 72 Feet 93 Parts.

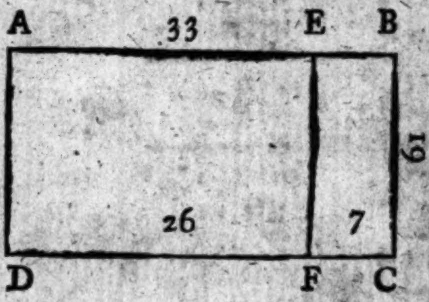
By Scale *and* Compaſſes.

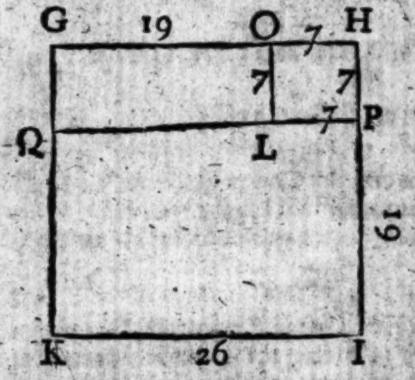
For the firſt *Example*, extend the Compaſſes from 12 to 15 Inches, (the Side of the Square) that Extent will reach from 18 Feet, (the Length, being twice turned over) to 28 Feet, and ſomething more.

For the ſecond *Example*, find a mean Proportional between 19 Inches and 33 Inches, by dividing the Space between them into two equal Parts, and the Compaſs Point will reſt upon 25, which is a mean Proportional between 19 and 33.

Then extend the Compaſſes from 12 to 25, (the Proportional found) that Extent will reach (being twice turned over) from 16.75 Feet, the Length, to 72.93 Feet, the Content.

A common Error is committed, for want of Art, in meaſuring theſe laſt Sorts of Solids, by adding the Depth and Breadth together, and taking Half for the Side of a mean Square. This Error, tho' it be but ſmall when the Depth and Breadth are pretty near equal, yet if the Difference be great the Error is very conſiderable; for the Piece of Timber, thus meaſured, will be more than the Truth, by a Piece, whoſe Length is equal to the Length of the Piece of Timber to be meaſured, and the Square equal to half the Difference of the Breadth and Depth, as I ſhall here demonſtrate.





I ſay, the Square GHIK is greater than the Parallelogram ABCD, by the little Square OHPL; for the Parallelogram QPIK is equal to the Parallelogram AEFD; and the Parallelogram GOLQ is equal to the Parallelogram EBCF. Therefore the Squa. is greater than the Parallelogram by the little Square OHPL. Which was to be prov'd.

Otherwiſe, you may prove it by Numbers, thus; the Sum of 33 and 19 is 52; the Half thereof is 26; the Square of 26 is 676; and the Product of the Depth and Breadth is 627; the Difference of theſe two is 49, equal to the Square of half the Difference; for the Difference between 33 and 19 is 14, the Half thereof is 7, whoſe Square is 49. Which was to be proved.

Now, if this 49 be multiplied by the Length of the Piece, and that Product divided by 144, to bring it to Feet, and thoſe Feet added to the true Content, the Sum will be equal to the Content found by the falſe Way mentioned.

See the **Work** of both.

33 Depth.
19 Breadth.

52 Sum.

26 Half.
26

156
52

676
16.75

3380
4732
4056
676

144)11323.00(78.63

1143
910
460

28

16.75 the Length.
49 the Sq. of half Diff.

15075
6700

4)820.75(5.69

1007
1435

139

Feet.
To 72.93 the true Content.
Add 5.69 the Part ſuperfluous.

Sum 78.62 equal to the Content by the falſe Way.

By Feet *and* Inches.

```
F: I.                                                   F. I.
 0  7                                                    2  2
 0  7                                                    2  2
-----                                                   -----
 0  4  1                                                 4  4
16  9                                                       4  4
--------                                                --------
 5  5  4                                                 4  8  4
    3  0  9                                             16  9
-----------                                             --------
 5  8  4  9 Part ſuperfluous.                           75  1  4
72 11  2  3 true Content add.                            3  6  3
-----------                                             --------
78  7  7  0 = Cont. the falſe Way——FalſeContent.        78  7  7
```

To find how much in Length makes a Foot of any Squar'd Timber.

Always divide 1728 (the ſolid Inches in a Foot) by the Area of the Baſe, the Quotient is the Length of a Foot.

This Rule is general for all Timber, which is of equal Thickneſs from End to End, whether it be Square, Triangular, Multangular, or Round.

Example 1: If a Piece of Timber be 18 Inches ſquare, how much in Length will make a Foot ſolid?

```
  18
  18
 ----
 144
  18
 ----
 324)1728(5 1/3
     ----
     1620
      108
```

Anſwer. 5 Inches and one Third.

By

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 18, that Extent will reach from 18 to 324, the Square or Area of the Baſe; then extend from 324 to 1728, that Extent will reach down from 1 to 5 Inches and $\frac{1}{3}$ of an Inch.

Or thus: Extend the Compaſſes from 18 to 41.569, that Extent (turned twice over from 1) will at laſt fall upon 5 $\frac{1}{3}$, as before.

Note, That 41.569 is the ſquare Root of 1728.

Example 2. If a Piece of Timber be 22 Inches deep, and 15 Inches broad, how much in Length will make a Foot.

22
15

110
22

330)1728(5.23

780
1200

210

Anſwer. 5 Inches and 23 Parts.

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 15, that Extent will reach from 22 to 330; then extend from 330 to 1728, that Extent will reach from 1 to 5.23 Inches, the Length of a Foot.

There is a Line for this Purpoſe upon moſt ordinary Rules, with a little Table at the End, of all ſuch Numbers as exceed the Length of the Rule, ſuch as this annexed.

0	0	0	9	9	0	11	3	9	Inches.
144	36	16	9	5	4	2	2	1	Feet.
1	2	3	4	5	6	7	8	9	Side of Squ.

Here you ſee, if the Side of the Square be 1, the Length muſt be 144 Feet; if two Inches be the Side of the Square, it muſt be 36 Feet in Length, to make a ſolid Foot, *&c.*

If the Side of the Square be not in the little Table, you will find it upon the Line, thus; if the Side of the Square be 16 Inches, you will find it againſt 6 Inches and 7 Tenths, counted from the other End of the Rule.

Then, if you take the Length of a Foot from the Line of Inches with your Compaſſes, and run the Compaſſes along the Piece from End to End, you will find how many Feet are contained in that Piece; or you may cut off any Number of ſolid Feet that ſhall be deſir'd; but if the Sides of the Piece be unequal, find a mean proportional Number, as is before taught, by dividing the Diſtance upon the Line of Numbers into two equal Parts, thus; if the Breadth be 25 Inches, and the Depth 9 Inches, divide the Space upon the Line of Numbers into two equal Parts, and you will find the middle Point at 15, ſo is 15 Inches the geometrical mean Proportional ſought; then, if you look for 15 upon the Line above-mention'd, that 7 Inches, and a little above half, will be the Length of a Foot.

* * *

§ III. *Of unequal* Squar'd Timber.

By unequal ſquar'd Timber, I mean all ſuch as have unequal Baſes, that is, ſuch as is thicker at one End than at the other; and ſuch are moſt Timber Trees, when they are hewn, and brought to their Squares.

The uſual Way to meaſure ſuch Timber, is to take a Square about the Middle of the Piece, which they take to be a mean Square: This Way, when the Piece is pretty near as thick at one End as at the other, is ſomething near the Truth; but when there is a great Diſproportion between the Ends of the Piece, the Error is conſiderable. All ſuch Solids being the Fruſtums of Pyramids, the true Way of meaſuring them muſt be by Sect. VII. Chap. 2. I ſhall give an *Example* or two, which I will work both by the true and falſe Ways, whereby you will ſee the Difference.

Example 1. If a Piece of Timber be 25 Inches ſquare at the greater End, and 9 Inches ſquare at the leſſer End, and 20 Feet long, how many Feet of Timber is in that Tree?

25
9
Sum 34
Half 17 The Side of the Square in the Middle.
17
119
17
289
20
144)5780(40.13
0100
560
128

By Rule II. Sect. VII. Chap. 2.

25
9
225

25
9
16 Diff. of the Sides.
16
96
16
3)256 The Square.
85.3333
225
310.3333

Anſwer. 40.13 Feet by the falſe Way

310.3333
20

144)6206.6660(43.101

446
146
266

122

Anſwer. 43.101 Feet by the true Way; ſo that there is near three Feet difference.

By Scale *and* Compaſſes.

Extend from 1 to 9, that Extent will reach from 15 (the ſame Way) to 225, the Rectangle of the Sides of the two Baſes; then the Difference between the ſaid Sides is 16: Extend from 3 to 16, that Extent will reach from 16 to 85.333, a third Part of the Square; which added to 225, the Sum is 310.333, a mean Area: Then extend from 144 to 310.333, that Extent will reach from 20 (the Length) to 43.1 Foot (the Content) the true Way.

Extend the Compaſſes from 12 to 17 (the Side of the middle Square) that Extent will reach from 20 (the Length being twice turned over) to 40.1 Foot, the Content by the falſe Way.

Example 2. If a Piece of Timber be 32 Inches broad and 20 Inches deep at the greater End, and 10 Inches broad and 6 deep at the leſſer End, and 18 Feet long, how many Feet of Timber are in that Piece?

Rule I. Sect. VII. Chap 2.

32
20

640
60

38400

6
10

60

38400

38400 . . . . (195.959 Mean Proportional.
1
29)284
385)2300
3909)375000
39185)231900
391909)3597500

640. The greater Base.
60. The lesser Base.

895.959 The Sum.
$6 = \frac{1}{3}$ Height.

5375.754

144)5375.754(37.33
1055
477
455

23

Add $\left\}\begin{matrix}32\\10\end{matrix}\right.$

Sum. 42

Half. 21
13

63
21

273 Area in the Middle.
18 Length.

2184
273

144)4914(34.12
594
180
360

72

$\left.\begin{matrix}20\\6\end{matrix}\right\}$ Add.

26 Sum.

13 Half.

Feet.
Answer. } Content the true Way 37.33
Content the false Way 34.12

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 10, that Extent will reach from 32 to 640, the Area of the greater Baſe.

Then extend from 1 to 10, that Extent will reach from 6 to 60, the Area of the leſſer Baſe. Then extend from 1 to 60, that Extent will reach from 640 to 38400, the Product of the two Areas: Find the Square Root thereof, by dividing the Space between 1 and 38400 into two equal Parts, ſo you will find the middle Point at 195.959, the Root ſought; which is a mean Proportional between the greater and the leſſer Areas. Then add the mean Proportional and two Areas together, and the Sum is 895.959; which multiplied by 6 (a third Part of the Length) by extending from 1 to 6, that Extent will reach from 895.959 to 5375.75. Then extend from 144 to 5375.75, and that Extent will reach from 1 to 37.33 Feet, the true Content.

For this falſe Way, half the Sum of the Breadth is 21, which is the Breadth in the Middle; and half the Sum of the Depth is 13: Extend from 1 to 13, that Extent will reach from 21 to 273, the Area of the middle Baſe. Then extend from 144 to 273, that Extent will reach from 18 (the Length) to 34.12, the Content the falſe Way.

§ IV. *Of* Round Timber, *whoſe Baſes are equal.*

THE uſual Way to meaſure round Timber Trees, is to girt them about the Middle with a String, and take the fourth Part of that Girt for the Side of a Square, by which they meaſure the Piece of Timber as if it was ſquare.

But that this is an Error, I ſhall make appear as follows: If the Circumference of a Circle be 1, the Area will be .07958; then the fourth Part of 1 is .25, which ſquar'd makes .0625. this they take for a mean Area inſtead of .07958: Therefore the true Content always bears ſuch Proportion to the Content found by the aforeſaid cuſtomary falſe Way, as .07958 to .0625; which is nearly as 23 to 18, ſo that in meaſuring by that cuſtomary falſe Way, there is above the one fifth Part loſt of what the true Content ought to be.

This Error, tho' it has been ſo often confuted, yet it is grown ſo cuſtomary in all Places, that there is little Hopes of my prevailing with Men that are ſo wedded to it, to embrace the Truth; I ſhall therefore, in the following *Examples*, ſhew how to work both the true Way, and alſo the falſe or cuſtomary Way.

Example 1. If a Piece of Timber be 96 Inches in Circumference, or Girth, and 18 Feet long, how many Feet of Timber is contained therein?

```
A fourth Part of 96 is 24
                       24
                      ----
                       96
                      48
                      ----
                      576 Area Baſe.
                       18
                      ----
                     4608
                     576
                     ----              Or thus.
                                        F.  I.
                144)10368(72            2   0
                    1008                2   0
                    -----              ------
                     288                4   0
                     288               18
                     ----              ------
                     ...               72   0
```

Content the falſe Way 72 Feet.

Then the true Way.

96
96
———
576
864
———
9216
.07958
———
73728
46080
82944
64512
———
733.40928 The Area by Prob. 5. Sect. IX. Ch. 1.
18
———
586727424
73340928
———
144)13201.36704(91.67
———
241 The true Content 91.67 Feet.
973
1096
———
88

By Scale *and* Compaſſes.

Extend from 12 to 24 (the fourth Part of the Girth) that Extent turned twice over from 18 Feet (the Length) will at laſt fall upon 72 Feet, the Content the cuſtomary Way.

Extend from 42.54 to 96, (the Girth) that Extent will reach from 18 Feet turned twice over, to 91.67 Feet, the true Content.

Example 2. If a Piece of Timber be 86 Inches girth, and 20 Feet long, how many Feet are contained therein?

The fourth Part of 86 is 21.5
21.5

1075
215
430

462.25
20

144)9245.00(64.2

605
290

20

F. I. P.
1 9 6
1 9 6

1 9 6
1 4 1 6
0 10 9

3 2 6 3
20

64 2 5 0

The Content the falſe Way 64.2 Feet.

By the true Way.

86
86

516
688

7396

```
        7396
      .07958
      ------
       59168
      36980
     66564
    51772
    ---------
    588.57368
           20
    ---------
144)11771.47360(81.74
    ---------
    251
    1074
     667
    ----
      91
```

The true Content 81.74 Feet.

By Scale *and* Compaſſes.

Extend from 12 to 21.5, that Extent turned twice over from 20, will reach at laſt to 64.2 Feet, the Content the falſe Way.

Extend from 42.54 to 86, that Extent turned twice over from 20, will at laſt fall upon 81.74 Feet, the true Content.

Theſe cylindrical Proportions may be very eaſily wrought upon the Line of Numbers.

Problem 1. Having the Diameter of a Cylinder in Inches, to find the Length of a Foot.

Suppoſe the Diameter 22.6 Inches.

As 22.6 : to 46.9 : : ſo is 1 to a fourth Number, and that to the Length of a Foot in Inches, 4.3.

Extend the Compaſſes from 22.6 to 46.9, that Extent will reach from 1 to a fourth Number; then turn them over again, and that will reach to 4.3 Inches.

Problem 2. Having the Diameter in Foot-meaſure, to find the Length of a Foot in Foot meaſure.

Suppoſe the Diameter 1.88 Feet.

Then, as 1.88 : to 1.128 :: ſo is 1 : to a fourth Number; and ſo is that to the Length of a Foot in Foot-meaſure .358.

Extend the Compaſſes from 1.88 to 1.128, that Extent turned twice from 1, will reach to .358 Parts of a Foot.

Problem 3. Having the Circumference in Inches, to find the Length of a Foot in Inches.

Suppoſe the Circumference 71 Inches.

Then, as 71 : to 147.36 :: ſo is 1 to a fourth Number, and ſo is that to the Length of a Foot in Inches 4.3.

Extend the Compaſſes from 71 to 147.36, that Extent turn'd twice from 1, will reach to 4.3 Inches, the Length of a Foot.

Problem 4. Having the Circumference in Foot meaſure, to find the Length of a Foot in Foot-meaſure.

Suppoſe the Circumference 5.92 Feet.

Then, as 5.92 : to 3.545 :: ſo is 1 : to a fourth Number; and ſo is that to the Length of a Foot in Foot-meaſure .358.

Extend the Compaſſes from 5.92 to 3.545, that Extent turn'd twice over from 1, will fall upon .358 Parts of a Foot.

Problem 5. Having the Diameter in Inches, and the Length in Inches, to find the Content in Inches.

Suppoſe the Diameter is 22.6 Inches, and the Length is 156 Inches, or 13 Feet.

Then as 1.128 : to 22.6 :: ſo is 156 : to a fourth Number; and ſo is that to the Content in Inches 62674.

Extend the Compaſſes from 1.128 to 22.6, that Extent turned twice from 156, will fall upon 62674 Inches, the Content.

Note, That 1.128 is the Diameter when the Side of the Square equal is 1.

Problem 6. Having the Diameter in Foot-meaſure, and Length in Feet, to find the Content in Feet.

Suppoſe the Diameter 1.88 Feet, and the Length 13 Feet.

Then, as 1.128 : to 1.88 :: ſo is 13 to a fourth Number; and ſo is that to the Content in Feet 36.27.

Extend from 1.128 to 1.88, that Extent turned twice from 13, will fall upon 36.27.

Problem 7. Having the Diameter in Inches, and Length in Inches, to find the Content in Feet.

Suppoſe the Diameter 22.6 Inches, and the Length 156 Inches.

Then, as 46.9 : is to 22.6 :: ſo is 156 : to a fourth Number; and ſo is that to the Content in Feet 36.27.

Extend from 46.9 to 22.6, that Extent turned twice from 156, will fall upon 36.27 Feet, the Content.

Note, That 46.9 is the Diameter of a Circle whoſe Area is 1728.

Problem 8. Having the Diameter in Inches, and Length in Feet, to find the Content in Feet.

Suppoſe the Diameter 22.6 Inches, and the Length 13 Feet.

Then, as 13.54 : to 22.6 :: ſo is 13 to a fourth Number; and ſo is that to the Content in Feet 36.27.

Extend from 13.54 to 22.6, that Extent turned twice from 13, will fall upon 36.27.

Note, That 13.54 is the Diameter of a Circle when the Area is 144.

Problem 9. Having the Circumference in Inches, and Length in Inches, to find the Content in Inches.

Suppoſe the Circumference 71, and the Length 156 Inches.

Then, as 3.545 : is to 71 :: ſo is 156 to a fourth Number; and ſo is that to 62674, the Content in Inches.

Extend the Compaſſes from 3.545 to 71, that Extent turned twice from 156, will fall upon 62674, the Content.

Note, That 3.545 is the Circumference when the Side of the Square equal is 1.

Problem 10. Having the Circumference in Feet, and Length in Feet, to find the Content in Feet.

Suppoſe the Circumference 5.92 Feet, and Length 13 Feet.

Then, as 3.545 : to 5.92 :: ſo is 13 : to a fourth Number; and ſo is that to 36.27.

Extend from 3.545 to 5.92, that Extent turned twice from 13, will fall upon 36.27 Feet, the Content.

Problem 11. **Having the Circumference in Inches, and Length in Inches, to find the Content in Feet.**

Suppoſe the Circumference 71 Inches, and Length 156 Inches.

Then, as 147.36 : to 71 :: ſo is 156 to a fourth Number; and ſo is that to the Content in Feet 36.27.

Extend the Compaſſes from 147.36 to 71. that Extent turn'd twice from 156, will fall upon 36.27 Feet, the Content.

Note, That 147.36 is the Circumference of a Circle whoſe Area is 1728.

Problem 12. **Having the Circumference in Inches, and Length in Feet, to find the Content in Feet.**

Suppoſe the Circumference 71 Inches, and Length 13 Feet.

Then, as 42.54 : is to 71 :: ſo is 13 : to a fourth Number; and ſo is that to the Content in Feet 36.27.

Extend the Compaſſes from 42.54 to 71, that Extent turned twice from 13, will reach to 36.27 Feet, the Content.

Note, That 42.54 is the Circumference of a Circle whoſe Area is 144.

§ V. *Of* Round Timber, *whoſe Baſes are unequal.*

THE uſual Way to meaſure round Timber (as I ſaid before) is to take a fourth Part of the Girth in the Middle of the Piece, for the Side of a mean Square. But this Way I have proved to be erroneous in Timber that is all the way of an equal Thickneſs; and it muſt be much more ſo in Timber that is tapering, and the more tapering it is the greater is the Error: For to the Error in the laſt Section, there is added the Error in the third Section; therefore, to meaſure all ſuch Timber according to Art and Truth, ſuch a Piece ought to be conſidered as a Fruſtum of a Cone, and ſhould be meaſured by the Rules given in Sect. VIII. Chap. II. by which Rules the following *Examples* are wrought.

Example 1. If a Piece of Timber be 9 Inches Diameter at the leſſer End, and 36 Inches at the other End, and 24 Feet long, how many Feet of Timber is therein?

```
      36              36 } Subtract.
       9               9 }
     ----            ----
Rect. 324              27 Difference.
                       27
                     ----
                      189
                      54
                     ----
                    3)729 The Square.
                     ----
                      243 One Third.
                      324 Rectangle add.
                     -----
                      567
                     -----
                    .7854
                      567
                    ------
                    54978
                   47124
                  39270
                  --------
     A mean Area 445.3218
                       24
                 ---------
                 17812872
                 8906436
                ----------
          144)10687.7232(74.22
                 607
                 317
                  292
                 ----
                   4
```

Anſwer. 74.22 Feet.

Or thus, by Feet and Inches.

F. I. F. I.
3 0 2 3 Difference.
0 9 2 3

2 3 Rect. 4 6
0 6 9

5 0 9 the Square.

1 8 3 one Third.
2 3 0 Rectangle added.

3 11 3 A mean Square.

F. I. P.
Then, as 14 : to 11 :: ſo is 3 11 3 to the Area.
11

7)43 3 9

2) 6 2 3

3 1 1 6
6

18 6 9 0
4

74 3 0 0

Here, inſtead of dividing by 14, I divide by 7 and by 2, becauſe twice 7 is 14.

And inſtead of multiplying by 24 Feet, the Length, I multiply by 6 and by 4, becauſe 6 times 4 is 24.

By Scale and Compaſſes this is too troubleſome.

Example 2. If a Piece of Timber be 136 Inches Circumference at one End, and 32 Inches Circumference at the other End, and 21 Feet long, how many Feet of Timber are contained in that Piece?

```
 136             136
  32              32
 ----            ----
 272             104 Difference.
408              104
 ----            ----
4352             416
                104
               ------
             3)10816 the Square.
              -------
              3605.333 one Third.
              4352     Rectangle add.
             --------
              7957.333 a mean Circumf. ſquar'd.
                .07958
             --------
              63658664
             39786665
            71615997
           55701331
           -----------
           633.24456014 the mean Area.
                     21
          ------------
          633324456014
         126648912028
         -------------
         13298.13576294

    144)13298.13(92.34
          ------
           338
            501
             693
            ----
             117
```

Anſwer. 92.34 Feet.

Bb

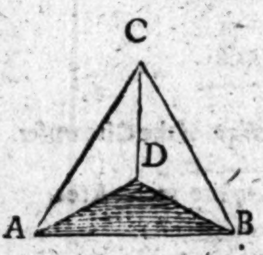
By Feet *and* Inches *thus*.

```
 F.  I.          F.  I.
 11  4           8   8    Difference.
  2  8           8   8
 ---------      ----------
 22  8           69  4
  7  6  8         5  9  4
 -----------    -----------
 32  2  8     3)75   1  4 The Square,
                -----------
                25  0  5  4
                30  2  8  0
                ------ -------
                55  3  1  4 The Squ. of the Circumfer.
                    F  I  P  S
As 88 : to 7 :: 55  3  1  4 : to the mean Area.
                          7
                  ------ -------
             11)386  9  9  4
                  ------ -------
              8) 35  1 11  9
                  ------------ ------
                   4  4  8 11  The mean Area.
                          7
                  ------ -------
                  30  9  2  5
                          3
                  ----------- -----
           Facit  92  3  7  3
```

§ VI. *Of the five* Regular Bodies.

THESE Bodies may all be meaſured by the fourth Section of Chap. II except it be the Cube, or Hexaedron, which is already meaſured in Sect. I. of that Chapter.

1. *Of the* TETRAEDRON.



A Tetraedron is a Solid contained under four equal and equilateral Triangles.

Let ABCD be a Tetraedron, whoſe Side is 12 Inches, the perpendicular Height 9.798 Inches.

By Sect. V. Chap. I. the Area of the Triangle will be found 62.352; a third Part of it is 20.784, which multiplied by the perpendicular Height, the Product is 203.641632 ſolid Inches, the Content.

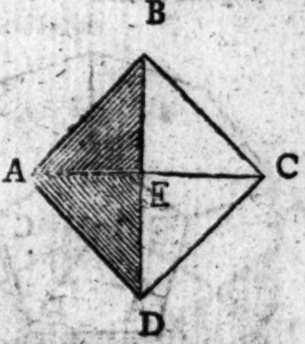
```
 10.392 The Perpendicular of the Triangle.
      6 Half the Side.
 ------
 62.352 Area of the Triangle.
 ------
 20784 One third Part.
  9.798 The Perpendicular Height.
 ------
 166272
187056
145488
187056
 ------
203.641632 The Solidity.
```

The ſuperficial Content is four times the Area of the Triangle, *viz.* 249 408 Inches,, becauſe there are 4 Triangles.

2. *Of the* OCTAEDRON.

The Octaedron is a Body contained under eight equal and equilateral Triangles.

Let ABCDE be an Octaedron whoſe Side is 12 Inches; the Content ſolid and ſuperficial is required.



An Octaedron is compoſed of two quadrangular Pyramids joined together by their Baſes; therefore, if the Area of the Baſe be multiplied into a third Part of the Length of both Pyramids, the Product will be the ſolid Content,

5.6568 A third Part of the Length.
144 Area of the ſquare Baſe.

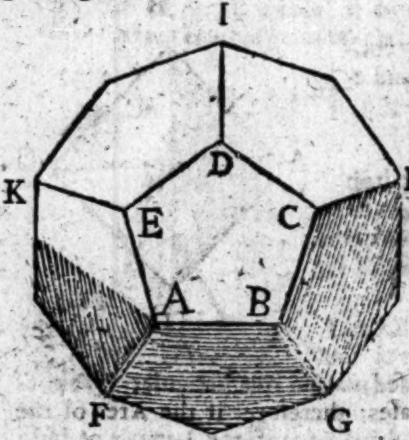
226272
226272
56568

814.5792 The Solidity.

The ſuperficial Content will be juſt double to that of the Tetraedron, *viz.* 498.816, becauſe the Side of this is ſuppoſed to be equal to the Side of that, and becauſe the Octaedron is contained under eight Triangles, and the Tetraedron but under four.

3. *Of the* DODECAEDRON.

The Dodecaedron is a ſolid Body, contained under twelve pentangular Planes.



Let ABCDEFGH IK be a Dodecaedron each Side thereof being 12 Inches; the Content ſolid and ſuperficial is required.

The Solidity of the Dodecaedron is compoſed of 12 pentangled Pyramids, whoſe Vertices all meet in the Center. Therefore if we find the Solidity of one of thoſe Pyramids, and multiply that by 12, that Product will be the Solidity of the Dodecaedron.

The Aliitude of one of the pentangled Pyramids will be found to be 13.36219.

The Perpendicular of the Pentagon will be

8.258292
30 Half Sum of the Sides.

247.748760 Area of the Pentagon.
60454.4 A third Part of 13.36219 inverted.

99099504
9909950
1238744
99099
1486

1103.48783 Content of one Pyramid.
12

13241.85396 The Solidity of the Dodecaedron.

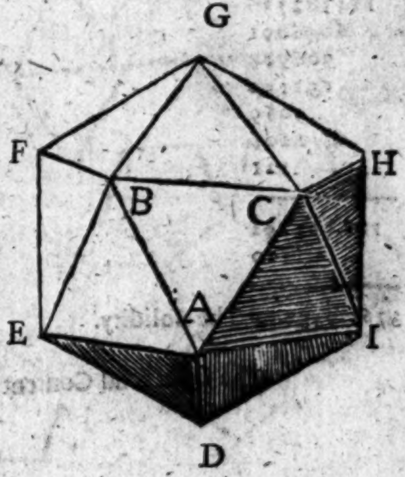
If the Area of the Pentagon be multiplied by 12, the Product will be the ſuperficia Content.

247.74876
12
————
2972.98512 The ſuperficial Content.

4. *Of the* ICOSAEDRON.

The Icoſaedron is a ſolid Body, contained under 20 equal and equilateral Triangles.

Let ABCDEFGHI be an Icoſaedron, each Side thereof being 12 Inches; the Content ſolid and ſuperficial is required.



The Icoſaedron is compoſed of twenty triangular Pyramids, with their Vertices all joining in the Center.

Therefore, if the ſolid Content of one Pyramid be multiplied by 20, the Product is the whole ſolid Content of the Icoſaedron.

10.39224 The Perpendicular of the Triangle.
6 Half the Side,

62.35344
20

1247.06880

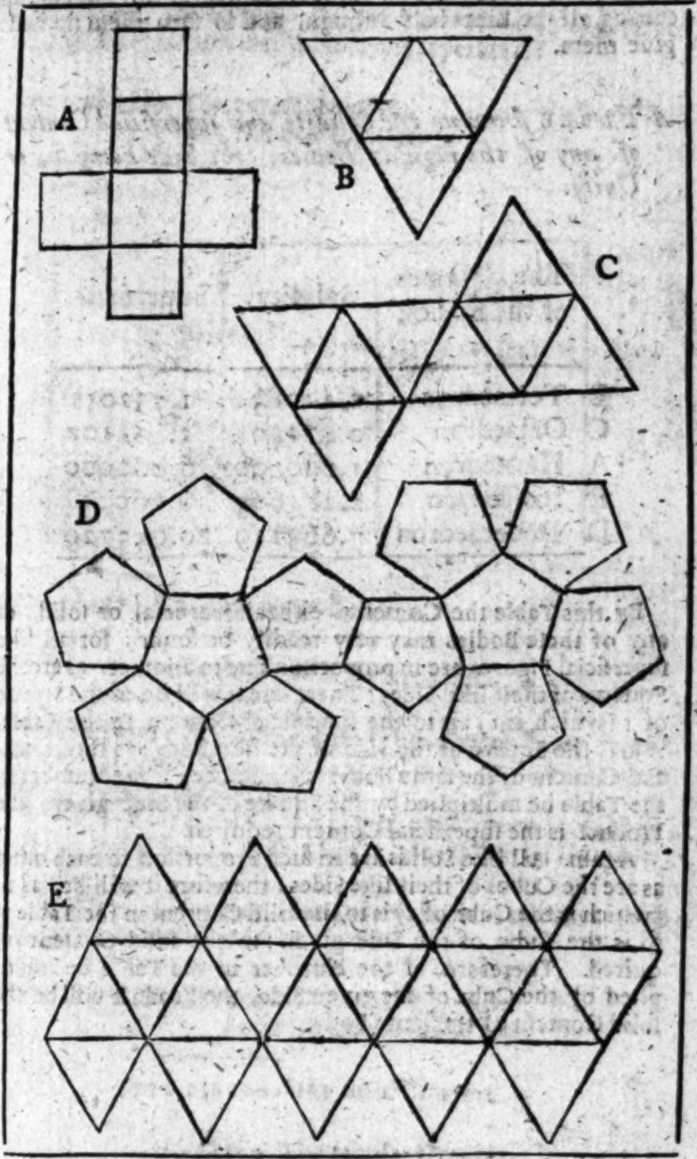
3.0230456 The third Part of the Altitude of the Pyramid.
44353.26

181382736
6046091
906914
151152
9069
1209
121

188.497292
20

3769.945840 The Solidity.

The ſuperficial Content 1147.0688.



A the Cube. B Tetraedron C Octaedron. D Dodecaedron. E Icoſaedron.

By theſe Figures you may cut theſe Bodies in fine Paſtboard, cutting all the Lines half through, and ſo turn them up and glue them.

A Table *ſhewing the Solidity and ſuperficial Content of any of the regular Bodies, the Side being* 1, *or Unity.*

	The Names of the Bodies.	Solidity.	Superficies.
B	Tetraedron	0,117851	1.732051
C	Octaedron	0.471404	3.464102
A	Hexaedron	1.000000	6.000000
E	Icoſaedron	2.181695	8.660254
D	Dodecaedron	7.663119	20.645729

By this Table the Content, either ſuperficial or ſolid, of any of theſe Bodies may very readily be found; for all like ſuperficial Figures are in proportion one to another, as are the Squares of their like Sides: Therefore it will be, as the Square of 1 (which is 1) : is to the ſuperficial Content in the Table, :: ſo is the Square of the Side of the like Body : to the ſuperficial Content of the ſame Body. Therefore, if the Number in the Table be multiplied by the Square of the Side given, the Product is the ſuperficial Content required.

Again. All like Solids are in ſuch Proportion to each other as are the Cubes of their like Sides; therefore it will be, as 1 : (which is the Cube of 1) is to the ſolid Content in the Table :: ſo is the Cube of the Side given : to the ſolid Content required. Therefore, if the Number in the Table be multiplied by the Cube of the given Side, the Product will be the ſolid Content of the ſame Body.

Example 1. If the Side of a Dodecaedron be 12 Inches (as before) what is the Content ſolid and ſuperficial?

7.663119 The tabular Number.
1728 The Cube of the Side.

61304952
15326238
53641833
7663119

13241.869632 The ſolid Content nearly the ſame as before.

20.645729 The tabular Number.
144 The Square of the Side.

82582916
82582916
20645729

2972.984976 The ſuperficial Content.

By Scale *and* Compaſſes.

Extend from 1 to 12, (the Side,) that Extent being turned three times over from 7.663119, will at laſt fall upon 13241.86, *&c.* the ſolid Content.

And if you apply the ſame Extent twice from 20.645729, it will at laſt fall upon 2972.98, *&c.* the ſuperficial Content.

Example 2. If the Side of an Octaedron be 20 Inches, what is the Content ſolid and ſuperficial?

.4714045 The tabular Number.
8000 The Cube of the Side.

3771.2360000 the ſolid Content.

3.464102 The tabular Number.
400 The Square of the Side.

1385.640800 The ſuperficial Content

By

By Scale *and* Compaſſes.

Extend from 1 to 10, that Extent, turned three times over from .4714045, will at laſt fall upon 3771.236, the ſolid Content. The ſame Extent turned twice over from 3.464, *&c.* will at laſt fall upon 1385.64, the ſuperficial Content.

§ VII. *How to meaſure any irregular Solid.*

IF you have any Piece of Wood or Stone that is craggy and uneven, and you deſire to find the Solidity, put the Solid into any regular Veſſel, as a Tub, a Ciſtern, or the like, and pour in as much Water as will juſt cover it; then take out the Solid, and meaſure how much the Fall of the Water is, and ſo find the Solidity of that Part of the Veſſel.

Example. Suppoſe a Piece of Wood or Stone to be meaſur'd, and ſuppoſe a Tub 32 Inches diameter, into which let the Stone or Wood be put and covered with Water; then when the Solid is taken out, ſuppoſe the fall of the Water 14 Inches; ſquare 32, and multiply the Square by .7854, the Product will be 804.2496, the Area of the Baſe; which multiplied by 14, the Depth or Fall of the Water, and the Product is 11259.49, *&c.* which divided by 1728, the Quotient is 6.51 Feet; and ſo much is the ſolid Content required.

CHAP. V.

Practical Queſtions in MEASURING.

Queſtion 1. IF a Pavement be 47 Feet 9 Inches long, and 18 Feet 6 Inches broad, I demand how many Yards are contained therein?

```
 F.   I.
 47   9                 47.75
 18   6                  18.5
 ___________           ________
 376  0                 23875
 47                     38200
   23 10  6             4775
    9  0  0            ________
    4  6  0           9)883.375
 ___________              98.1
9)883 4  6
```

Anſwer. 98 Yards 1 Foot.

Queſt. 2. There is a Room whoſe Length is 21.5 Feet, and the Breadth 17.5 Feet, is to be paved with Stones each 18 Inches ſquare; I demand how many ſuch Stones will pave it?

```
       21.5              1.5
       17.5              1.5
      ------            -----
       1075               75
      1505               15
      215               -----
    --------            2.25 Area of one Stone.
2.25)376.25(167
     ------
      1512
       1625
      -----
        50
```

Anſwer. 167 Stones.

Queſt. 3. There is a Room 109 Feet 9 Inches about, and 9 Feet 3 Inches high, which is all (except two Windows, each 6 Feet 6 Inches high, and 5 Feet 9 Inches broad) to be hung with Tapeſtry that is Ell broad; I deſire to know how many Yards will hang the ſaid Room.

From the Content of the Room ſubtract the Content of the Windows, and divide the Remainder by the ſquare Feet in a Yard of Tapeſtry.

```
 3.75        109.75 Length.                                 5.75
    3          9.25 Breadth.                                6.5
 -----     ---------                                       -----
11.25         54875                                         2875
             21950                                          3450
            98775                                          -----
          ---------                                         37375
          1015.1875 Content of the Room.                        2
            74.75   Content of the Windows ſubt.           -----
      ---------------                                      74.75
  11.25)940.4375(83.59
        ---------
         4043
          6687
          10625
         -------
          500
```

Anſwer. 83.59 Yards.

Queſt. 4. If the Axis of a Globe be 27.5 Inches, I demand the Content ſolid and ſuperficial.

3.1416
27.5

157080
219912
62832

86.39400 The Circumference.
27.5 The Diameter.

431970
604758
172788

6)2375.8350 The ſuperficial Cont.

395.9725 A ſixth Part.
27.5

19798625
27718075
7919450

10889.24375 The Solidity in Inches.
Anſwer { 6.3 Feet ſolid.
16 49 Feet ſuperficial.

Queſt. 5. There is the Fruſtum of a Globe, the Diameter of whoſe Baſe is 24 Inches, and the Altitude thereof is 10 Inches; what is the Content ſolid and ſuperficial?

Find the Superficies, as is directed in Page 169, and find the Solidity by the firſt Theorem in Page 180.

24	.7854	.7854	20
24	576	400	20
96	47124	314.16000	400
48	54978		
576	39270		
	452.3904 } add.		
	314.16		

766.5504

766.5504 The Curve-Superficies.
452.3904 The Baſe add.

1218.9408 The whole ſuperficial Content in Inches.

12×12=144
3

432
100 The Square of the Altitude add.

532 The Sum.
10 Multiply by the Altitude.
5320

.5336 Multiply

31920
15960
10640
26600

2785.5520 The Solidity in Inches.

Queſt. 6. If a Tree girt 18 Feet 6 Inches, and be 14 Feet long, how many Tuns of Timber are contained in that Tree?

F. I.
4)18 6 The Girt

4 7 6 A 4th Part.
4 7 6

18 6 0
2 8 4 6
2 3 9

21 4 8 3
6

128 4 1 6

Here I multiply by 6 and by 4,
because 6 times 4 is 24.

128 4 1 6
4

4|0)51|3 4 6 0

12 33

Anſwer. 12 Tuns, 33 Feet, 4 Inches, 6 Parts.

Note, That 40 Feet of Timber is a Tun, and 50 Feet a Load.
Note alſo, That 4 Feet broad, 4 Feet deep, and 8 Feet long is a Cord of Fire-wood, that is, 128 cubical Feet.

Queſt. 7. There is a Cellar to be dug by the Floor, whoſe Length is 33 Feet 7 Inches, and the Breadth is 18 Feet 9 Inches, and its Depth is to be 5 Feet 9 Inches; I demand how many Floors of Earth are in that Cellar?

F. I.
33 7 The Length.
18 9 The Breadth.

264
33
16 9 6
8 4 9
9 0 0
1 6 0

629 8 3
5 9 The Depth:

3148 5 3
314 10 1
157 5 0

324)3620 8 4(11

80

56

Anſwer. 11 Floors 56 Feet.

Note, That 18 Feet ſquare and a Foot deep is a Floor of Earth, that is 324 ſolid Feet

Queſt. 8. There is a Roof covered with Tiles, whoſe Depth on both Sides (with the uſual Allowance at the Eaves) is 35 Feet 6 Inches, and the Length 48 Feet 9 Inches; how many Squares of Tiling are contained therein?

F. I.
48 9
35 6

240
144
24 4 6
17 6 0
8 9 0

17|30 7 6

Anſwer. 17 Squares 30 Feet.

Queſt. 9. There is a Cone whoſe Diameter at the Baſe is 42 Inches, and the perpendicular Height 94 Inches, and it is required to cut off two ſolid Feet from the top End thereof; I demand what Length upon the Perpendicular muſt be cut off?

42
42

84
168

1764
.7854

7056
8820
14112
12348

1385.4456
94

55417824
124690104

3)130231.8864

43410.628

1728
2

3456

94
94

376
846

8836 Square.
94

35344
79524

830584 The Cube.

All the ſolid Bodies are in triplicate Reaſon of their homologous Sides by *Eucl.* 12, 12; 12, 18; and 11, 33; therefore it will be,

Sol. of the Cone. Cube Alt. Sol. of 2 Feet.
As 43410.6288 : 830584 :: 3456 :
3456

4983504
4152920
3322336
2491752

43410.6288)2870498304(66124 The Cube of the Length

265860576
5396803
1055740
187528

66124(40.43
64

2124000 Reſolvend.

12
48

492 Diviſor.

120
4800

48120 Diviſor.
64
1920
19200

[illegible]939264 Subtrahend.

184736000 Reſolvend.

184736000

```
    1212
  489648
  ———————
  4897692 Diviſor.
  ———————
       27
   10908
 1468944
 —————————
 147003507 Subtrahend
 —————————
  37732492
```

Anſwer. The Length upon the Perpendicular muſt be 40.43 Inches. If it had been 3 Feet, the Length had been 46.29 Inches.

If two Feet were to be cut off from the Bottom, or greateſt End, then from 43410.6288 ſubtract 3456, and the Remainder is 39954.6288. Then ſay,

```
As 43410.6288 : 830584 :: 39954.6288
                              830584
                          ——————————
                           1598185152
                          3196370304
                         1997731440
                        1198638864
                       3196370304
                       ——————————————
43410.6288)33185675407.2192(764459(91.4
    ....                               729
           —————————————                ———————
           279823524                     35459
            19359751                    ———————
             1995590                        27
              259075                       243
               42022                    ———————
           —————————                      2457
                2952
```

```
     2457
  ---------
      271
    243
  ---------
    24571
  ---------
   10888000
  ---------
       273
    24843
  ---------
    248703
```

Anſwer. It muſt be cut at 91.4 Inches from the Top, or 2.6 Inches from the Bottom.

Queſt. 10. If a ſquare Piece of Timber be 12 Feet long, and if the Side of the Square of the greater Baſe be 21 Inches, and the Side of the Square of the leſſer Baſe be 3 Inches; how far muſt I meaſure from the greater End to cut off 5 ſolid Feet?

Firſt, find the Length of the whole Pyramid, thus; the Difference between 21 and 3 is 18; then,

Diff. Length. great. Length.
As 18 : 12 :: 21 : 14.

So I find the whole Length of the Pyramid 14 Feet, or 168 Inches.

The ſolid Content of the whole Pyramid is 24696 Inches, and the ſolid Content of 5 Feet is 8640; which ſubtracted from 24696, there remains 16056 Inches. Then, the Cube of 168 (the Length) is 4741632. Then,

As 24696 : 4741632 :: 16056 :

To 3082752, whoſe Cube Root is 145.54; ſubtract this Root from 168 (the Length) and there remains 22 46 Inches, which is the Length of 5 ſolid Feet at the great End.

Queſt. 11. Three Men bought a Grinding Stone of 40 Inches Diameter, which coſt 20 Shillings: of which Sum the firſt Man paid 9 Shillings, the ſecond 6 Shillings, and the third 5 Shillings; I demand how much of the Stone each Man muſt grind down, proportionable to the Money he paid?

All Circles are in duplicate Reaſon of their Diameters, by *Eucl.* 12. 2.

Square the Semidiameter, which makes 400. Then,

$$\text{As } \overset{s.}{20} : 400 :: \overset{s.}{9} : 180.$$

This 180 is the Square of the Semidiameter of the Circle belonging to the firſt Man.

$$\text{And, as } \overset{s.}{20} : 400 :: \overset{s.}{6} : 120.$$

This 120 is the Square of the Semidiameter of the Circle belonging to the ſecond.

$$\text{And, as } \overset{s.}{20} : 400 :: \overset{s.}{5} : 100.$$

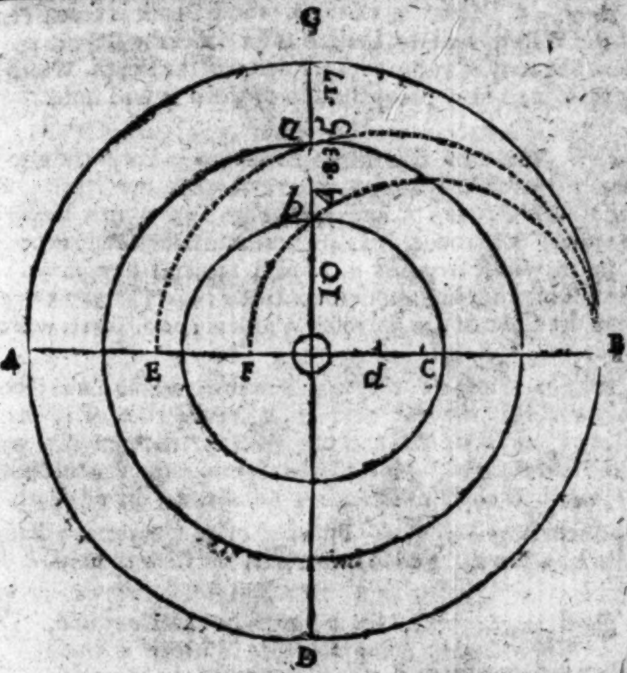
This 100 is the Square of the Semidiameter of the Circle belonging to the third.

Then, from 400, (the Square of the Semidiameter of the Stone) ſubtract 180, and there remains 220, whoſe ſquare Root is 14.83 Inches; which ſubtracted from 20 Inches (the Semidiameter) there remains 5.17 Inches, which is the Breadth of the Ring, or Part of the Stone which muſt be ground down by the firſt.

Then, from 220 ſubtract 120, and there remains 100, whoſe ſquare Root is 10: ſubtract that from 14.83, and there remains 4.83 Inches, the Breadth of the Ring, or Part to be ground down by the ſecond Man. The third muſt grind down the Remainder, which is 10 Inches, the ſquare Root of 100.

This Queſtion may very eaſily and ſpeedily be performed geometrically, as in the annexed Scheme.

First,



First, upon the Center ⊙ ſtrike the Circle ACBD, and croſs it at right Angles with the two Diameters AB and CD: Then divide the Semidiameter A⊙ (which ſuppoſe 20) in Proportion, as 9 *s.* 6 *s.* and 5 *s.* (the ſeveral Sums paid by the three Men) by the Points E and F; ſo ſhall AE be 9, EF 6, and F⊙ 5: Then divide EB into two equal Parts in *d*, and upon *d*, as a Center, ſtrike the Semicircle E*a*B, and divide FB into two equal Parts in *c*, and upon *c*, as a Center with the Radius *c*F, ſtrike the Semicircle F*b*B: So have you the Semidiameter ⊙C divided into three ſuch Parts as the Stone ought to be divided; and Circles ſtruck thro' theſe Points will ſhew how much each Man muſt grind for his Share.

Queſt. 12. A Gard'ner he had an upright Cone,
Out of which ſhould be cut him a Rolling-ſtone,
The biggeſt that e'er it could make;
The Maſon he ſaid, that there was a Rule
For ſuch ſort of Work, but he had a thick Skull,
Now help him for Pity's Sake.

Anſwer. It muſt be cut at one third Part of the Altitude.

Queſt. 13. There is a Ciſtern whoſe Depth is ſeven Tenths of the Width, and the Length is ſix times the Depth, and the ſolid Capacity is 367.5 Feet; I demand the Depth, Width and Length, and how many Buſhels of Corn it will hold.

Firſt, you muſt find three Numbers in Proportion to the Depth, Width and Length, thus; ſuppoſe the Depth 7, then the Width will be 10, and the Length 42; which multiplied together, the Product is 2940, which are the ſolid Inches in a Ciſtern, whoſe Depth is 7, Width 10, and Length 42. But the ſolid Inches in the Queſtion are 635040 (= 367.5 × 1728) then the Cube of the ſuppoſed Width is 1000. So it will be,

As 2940 : 1000 :: 635040 : 216000, whoſe Cube Root is 60, which is the true Width; 7 Tenths thereof is 42, the Depth; and 6 times 42 is 252 Inches, the Length; which three Numbers being multiplied together, the Product will be 635040. If theſe ſolid Inches be divided by 2150.42, the Quotient is 295 $\frac{66610}{215042}$ Buſhels, or 36 Quarters, 7 Buſhels, 1 Peck, 4 Pints. And ſo much will the Ciſtern hold.

Queſt. 14. Suppoſe, Sir, a Buſhel be exactly round,
Whoſe Depth, being meaſur'd 8 Inches is found,
If the Breadth 18 Inches and half you diſcover,
This Buſhel is legal all *England* over.
But a Workman would make one of another Frame,
Seven Inch and a half muſt be the Depth of the ſame:
Now, Sir, of what Length muſt the Diameter be,
That it may with the former in Meaſure agree?

18.5

```
      18.5
      18.5
    ------
       925
     1480
     185
    ------
    342.25 the Square.
     .7854
    ------
    1369000
   1711125
  2738000
 2395750
 ---------
 268.8031500
           8
 ---------
2150.425200 the ſolid Inches in a Buſhel.
```

```
7.5)2150.4252(286.72336
      . . .
    -------
    650
     504
      542
       175
        252
         270
          450
          ---
           0
```

.7854)286.72336...)365.0666(19.107
1

51103 29)265
39793 261
52360
5236 381)406
381

38207)256600

Anſwer. The Diameter muſt be 19,107 Inches, if the Depth be 7.5 Inches.

Queſt. 15. *In the midſt of a Meadow well ſtored with Graſs,*
I took juſt an Acre to tether my Aſs:
How long muſt the Cord be, that feeding all round,
He mayn't graze leſs nor more than his Acre of
Ground?

By *Problem* 10. Section IX. Chap. I. find the Diameter of a Circle containing an Acre; half that will be the Length of the Cord.

The Work.

660 Feet, the Length of an Acre.
66 Feet, the Breadth of an Acre.

3960
3960

43560 the ſquare Feet in an Acre.

As 1 : 1.2732 :: 43560
43560

763920
63660
38196
50928

```
  .  .  .
  55460.5920(235.5 Diamet.
  4           --------
  -------     117.75 Half.
43)154
   129
  ------
465)2560
    2325
  -------
4705)23559
     23525
  ---------
        34
```

Anſwer. The Cord muſt be 117 Feet and 9 Inches.

But in an *Iriſh* Acre is 70560 Feet, *i. e.* 21 × 21 × 160 = 70560; then ſay as above.

```
As 1 : 1.2732 :: 70560
           70560
         --------
         7633920
         63660
        891240
        ---------
  .  .  .  .  .
  89836.9920(299.72 Diameter.
  4           ---------
  ---------   199.86 Half.
49)498
   441
  --------
589)5736
    5301
  ---------
5987)43599
     41909
  ----------
5994)169020
```

Queſtion 16. A Maltſter has a Kiln that is 16 Feet 6 Inches ſquare; but he is minded to pull it down, and build a new one, that may be big enough to dry three times as much at a time as the old one will do; I demand how much ſquare the new one muſt be?

```
        16.5
        16.5
      ------
         825
        990
       165
      ------
      272.25 the Area of the old one.
           3
      ------
      .  .  .
      816.75)28.57
      4
    48)416
       384
      ------
   565)3275
       2825
      ------
  5707)45000
       39949
      ------
        5051
      ------
```

Anſwer. The Side of the new one muſt be 28 Feet, and near 7 Inches.

Queſtion 17. If a round Ciſtern be 26.3 Inches Diameter, and 52.5 Inches deep, how many Inches Diameter muſt a Ciſtern be, to hold twice the Quantity, the Depth being the ſame? And how many Ale Gallons will each Ciſtern hold?

26.3
26.3

789
1578
526

691.69 the Square.
2

1383.38(37.19
9

67)483
469

741)1438
741

7429)69700
66861

2839

The Diameter of the greater is 37.19 Inches.

691.69 the Square of the leſſer Ciſtern's Diameter.
.7854

276676
345845
553352
484183

543.253326 Area of the Baſe.
52.5

2716266630
1086506652
2716266630

28520.7996150 Solid Content in Inches.

282)28520.799(101,137 Gallons.

320
387
1059
2139

165

NOTE, That 282 ſolid Inches is an Ale or Beer Gallon, and 231 a Wine Gallon.

And 359.05 is the Square of the Diameter of a Circle that will hold a Gallon of Ale at an Inch deep, and 294.12 for Wine.

And 217.6 Inches is an *Iriſh* Gallon, of either Ale or Wine. Alſo 277.05 is the Square of the Diameter of a Circle, that will hold an *Iriſh* Gallon at an Inch deep.

You may find the Content in Gallons, thus: Divide the Square of the Diameter by 359,05, and multiply the Quotient by the Depth.

359.05)

```
359.05)1383.38 . . . (3.853
       -------           52.5
       306230         -------
        18990           19265
         1037           7706
                       19265
                      -------
```

The Content of the greater 102,2825 Gallons.

Queſtion 18. If the Diameter of a Cask at the Bung be 32 Inches, and at the Head 25 Inches, and the Length 40 Inches, how many Ale Gallons is contained therein?

```
 25       32                                   359
 25       32                                     3
----     ----                                 -----
125       64                                  1077
 50       96
----     ----
625      1024 Square of the Bung Diameter.
         1024 the ſame.
          625 Square of the Head Diameter.
         ----
   1077)2673    (2.48          But for Iriſh Gallons,
        -----     40           Divide by         831.15
         5190   -----          The Quotient is     3.21
          8820  99.20          Which multip. by     40
        ------                                  -------
           204                 Give Iriſh Gallons 128.40
```

Anſwer. 99.2 Gallons.

Otherwiſe, you may find a mean Diameter, and work by Scale and Compaſſes, thus: Subtract 25 from 32, and there remains 7, which multiplied by .7, the Product is 4.9, which added to 25, the Sum is 29.9. Then extend the Compaſſes from 18.95, the Gage Point *Engliſh*, to 29.9 that Extent, turned twice from 40, (the Length) will fall upon 99.6 Gallons; ſomething more than before.

Queſtion 19. There is a Stone 20 Inches long, 15 Inches broad, and 8 Inches thick, which weighs 217 Pounds; I demand the Length, Breadth and Thickneſs of another of the ſame Kind and Shape, which weighs 1000 Pounds?

The Cube of 20 (the Length) is 8000. Then, (by *Euclid*, 11. 33.)

As 217 : 8000 :: 1000 : 36870.645, whoſe Cube Root is 33.28 Inches, the Length of the Stone weighing 1000 Pounds. Then ſay,

As 20 : 33.28 :: 15 : 24.96
As 20 : 33.28 :: 8 : 13.312

Anſwer.			
	The Length	33.28	Inches.
	The Breadth	24.96	
	The Thickneſs	13.312	

Queſtion 20. If an Iron Bullet, whoſe Diameter is 4 Inches, weighs 9 Pounds, what will be the Weight of another Bullet (of the ſame Metal) whoſe Diameter is 9 Inches?

The Cube of 4 is 64, and the Cube of 9 is 729. Then, (by *Eucl.* 12. 18.)

lb. *lb.*
As 64 : 9 :: 729 : 102.515

lb. *oz.* *dr.*
Anſwer. It weighs 102 8 4 *fere.*

Queſtion 21. There is a ſquare Pyramid of Marble, each Side of its Baſe is 5 Inches, and the Height thereof 15 Inches, and its Weight is 12 Pounds and a Quarter; I demand the Weight of another like ſquare Pyramid, each Side of whoſe Baſe is 30 Inches?

The Cube of 5 is 125, and the Cube of 30 is 27000. Then, (by *Eucl.* 12. 12.)

lb. *lb.*
As 125 : 12.25 :: 27000 : 2646

Anſwer. The Weight is 2646 Pounds.

Queſtion 22. There is a Ball or Globe of Marble, whoſe Diameter is 6 Inches, and its Weight 11 Pounds; what will be the Diameter of another Globe of the ſame Marble, that weighs 500 Pounds?

The Cube of 6 is 216. Then,

```
      lb.          lb.
As 11 : 216 :: 500 : 9818.1818
```

whoſe Cube Root is 21.4 Inches, the Diameter ſought.

Queſtion 23. There is a Fruſtum of a Pyramid, whoſe Baſes are regular Octagons; each Side of the greater Baſe is 21 Inches, and each Side of the leſſer Baſe is 9 Inches, and its Length is 15 Feet; I demand how many ſolid Feet are contained therein?

```
            4.8284 the tabular Number, Page 87.
              237 the Square of a mean Side.
      ------
      337988
     144852              21          12
     96568                9          12
    ---------           ----        ----
    1144.3308           189       3)144
           15            48         ----
    ---------           ----          48
     57216540           237
    11443308
    ---------
144)17164.9620(119.2
    ---------
    276
    1324
     289
       9
```

Anſwer. 119.2 ſolid Feet.

Queſtion 24. There is a Fruſtum of a Cone, the Diameter of the greater Baſe is 36 Inches, and the Diameter of the leſſer Baſe is 20 Inches, and the Length or Height is 215 Inches; I demand the Length and ſolid Content of the whole Cone and alſo the ſolid Content of the given Fruſtum?

First find the Length of the whole Cone, thus;

From 36
Subtr. 20

As 16 : 215 :: 36 : 483.71

ſo the Length of the whole Cone is 483 $\frac{3}{4}$ Inches.

Then find the Content of the whole Cone.

```
        36
        36
      ----
       216
       108
      ----
      1296
     .7854
     -----
      5184
     6480
   10368
   9072
   -------
Area Baſe 1017.8784
```

```
      1017.8784
        52.161
      ---------
      10178784
      6107270
       101788
        20357
         5089
      -------- Feet.
1728)164132.88(94.98
      --------
      8612
      17008
       14568
      -------
         744
```

Thus I find the Solidity of the whole Cone 94.98 Feet.

Then find the ſolid Content of the top Part that is wanting.

.7854 the Area of Unity.
400 The Square of 20.

3)314.1600 Area of the leſſer Baſe.

104.72 A third Part.
268.75 Altitude of the top Part.

52369
73304
83776
62832
29944

1728)28143.5000(16.28 Feet,

10863
4955
14990

1166

	Feet,
Content of the Whole	94.98
Content of the top Piece	16.28
Content of the Fruſtum	78.7

Queſtion 25. If the top Part of a Cone contains 26171 ſolid Inches, and 200 Inches its Length, and the lower Fruſtum thereof contains 159610 ſolid Inches; I demand the Length of the whole Cone, and the Diameter of each Baſe?

200
200

40000
200

8000000

159610 } add.
26171 }

185781 The Sum.

As 26171 : 8000000 :: 185781 :to 56789881, whoſe Cube Root is 384.3 Inches, the Length of the whole Cone.

Then find the Diameter of the leſſer Baſe, thus;

```
200)26171
    -----
   130.855
         3
   -------
   392.565 Area of the leſſer Baſe.
```

Then, by Prob. 10. Sect. IX. Chap. I.

```
As 1 : 1.2732 :: 392.565
                 1.2732
                -------
                 785130
               1177695
              2747955
              785130
             392565
            ----------
                 . . .
                499.8137580(22.35
            4
            --
            42)99
               84
            ----
            443)1581
                1329
            -------
            4465)25237
                 22325
            ---------
                  2912
```

Again, As 200 : 22.35 :: 384.3 : 42.94
Leſſer Leng. Leſſ. Diam. Greater Leng. Gr. Diam.

		Inches
Anſwer.	The Length of the whole Cone	384.3
	The Diameter of the greater Baſe	42.94
	The Diameter of the leſſer Baſe	22.35

Queſtion

Queſt. 16. There is a Fruſtum of a Cone whoſe ſolid Content is 20 Feet, and its Length 12 Feet; and the greater Diameter bears ſuch Proportion to the leſſer as 5 to 2; I demand the Diameters.

$5\times5=25$
$2\times2=\ \ 4$
$5\times2=10$

The Sum 39

3)12

4)20(5 Feet.

These 5 Feet are the Triple of a mean Area.

Then, as 1 : 1.27324 :: 5 : *6.3662*.

So the triple Square of a mean Diameter is *6.3662*.

Then, as 39 : *6.3662* :: 25 : 4.080897.

This 4.080897 is the Square of the greater Diameter, whoſe ſquare Root is 2.020123 Feet, which is 24.24147 Inches.

Then, as 5 : 24.24147 :: 2 : *9.69659*.

So the greater Diameter is 24.24147, and the leſſer Diameter is *9.69659* Inches.

Queſt. 27. There is a Room of Wainſcot 129 Feet 6 Inches in Circumference, and 16 Feet 9 Inches high, (being girt over the Mouldings) there are two Windows, each 9 Feet 3 Inches high, and the Breadth of each, from Cheek to Cheek, 5 Feet 6 Inches; the Breadth of the Shutters of each is 4 Feet 6 Inches; the Cheek-Boards and Top and Bottom Boards of each Window, taken together, is 24 Feet 6 Inches, and their Breadth 1 Foot 9 Inches; the Door-Caſe 7 Feet high, and 3 Feet 6 Inches wide; the Door 3 Feet 3 Inches wide; I demand how many Yards of Wainſcot are contained in that Room?

F. I.
129 6
16 9

782 0
129
64 9
32 4 6

2169 1 6

F. I.
3 3
7 0

22 9
11 4 6 Half.

34 1 6

F. I.
7 3
5 6

36 3
3 7 6

39 10 6
2

79 9 0

F. I.
7 3
4 6

29 0
3 7 6

32 7 6
16 3 9 Half.

48 11 3
2

97 10 6

F. I.
24 6
1 9

24 6
18 4 6

42 10 6
2

85 9 0

F. I.
3 6
7 0

$\left.\begin{array}{ll}24 & 6\\ 79 & 9\end{array}\right\}$ add.

104 3

The

The Content of the Room	2169	1	6
The Shutters, at Work and half	97	10	6
The Door, at Work and half	34	1	6
The Cheek-Boards, *&c.*	85	9	0
The Sum	2386	10	6
The Window-Lights and Door-Caſe deduct	104	3	0
	9)2282	7	6
	253	5	

Anſwer. 253 Yards, 5 Feet.

Queſt. 28. There is a Wall which contains 18225 Cube-Feet, and the Height is 5 times the Breadth, and the Length 8 times the Height; what is the Length, Breadth and Height?

Suppoſe the Breadth 2, then the Height muſt be 10, and the Length 80; which three Numbers multiplied together, the Product will be 1600, and the Cube of 2 is 8; then ſay,

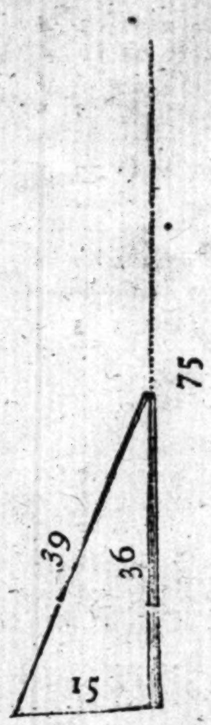
As 1600 : 8 :: 18225 : 91.125.

Then the Cube Root of 91.125 is 4.5, which is the Breadth; then 5 times 4.5 is 22.5, the Height, and 8 times 22.5 is 180, the Length.

Queſt. 29. There is a May-pole, whoſe Top End was broken off by a Blaſt of Wind, and the Top End in falling ſtruck the Ground at 15 Feet diſtance from the Foot of the May-pole; the broken Piece was 39 Feet; now I demand the Length of the May-pole.

By *Eucl.* 1. 47. the Square of the Hypothenuſe of a right-angled Triangle, is equal to the Sum of the Squares of the Baſe and Perpendicular.

Therefore, from the Square of 39 ſubtract the Square of 15, the ſquare Root of the Remainder is the Piece ſtanding, to which add the Piece broken off, and you have the whole Length.



39
39

351
117

1521
225

1296(36
9

66)396
396

15
15

75
15

225

The Piece ſtanding is 36 Feet.
The Piece broken off is 39 Feet.

The whole Length 75

Queſt. 30.

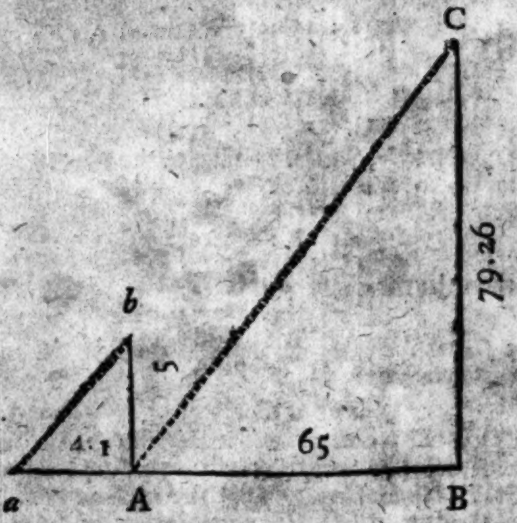
A May-pole there was, whoſe Height I would know;
The Sun ſhining clear, ſtrait to work I did go:
The Length of the Shadow, upon level Ground,
Juſt ſixty-five Feet, when meaſur'd, I found:
A Staff I had there, juſt five Feet in Length;
The Length of its Shadow was four Feet one Tenth:
How high was the May-pole, I gladly would know?
And it is the Thing you're deſir'd to ſhew.

By *Eucl.* 6. 4.
As aA : Ab :: AB : BC.

That is,

As 4.1 : 5 :: 65 : 79.26.

So I find the Height of the May-pole to be 79 Feet, and a little above three Inches.



Here AB repreſents the Length of the Shadow of the May-pole, and BC the May-pole; *a*A the Shadow of the Staff, and A*b* the Staff.

Queſt. 31. What will be the Diameter of a Globe, when the Solidity and ſuperficial Content thereof are equal?

If the Diameter be 1, the Solidity will be .5236, and the Superficies will be 3.1416; that is, as 1 to 6. And ſo find the ſuperficial Content, we muſt multiply 3.1416 by the Square of the Axis, or Diameter, and the Product is the ſuperficial Content. And for the Solidity, multiply .5236 by the Cube of the Axis, the Product is the ſolid Content: Therefore, becauſe .5236 is a ſixth Part of 3.1416, we muſt take ſix for the Diameter ſought. For if 3.1416 be multiplied by the Square of 6, *viz.* by 36, the Product will be 113.0976; and if .5236 be multiplied by the Cube of 6, *viz.* by 216, the Product is likewiſe 113.0976, the Solidity equal to the Superficies. [illegible] is the true Anſwer.

Queſt. 32. What will the Axis of a Globe be, when the Solidity is in Proportion to the Superficies, as 18 to 8?

Becauſe the Solidity and Superficies is as 1 to 6; when the Axis of the Globe is 1, it will be

As 8 : 18 :: 6 : 13.5.

So the Diameter ſought is $13\frac{1}{2}$.

If the Proportion of the Solidity to the Superficies had been as 8 to 18, then it will be

As 18 : 8 :: 6 : $2\frac{2}{3}$.

So then the Diameter will be $2\frac{2}{3}$.

The Reaſon of theſe Operations, both in this and the laſt Queſtion is from Algebra.

Queſt. 33. There are three Grenado Shells of ſuch Capacity, that the ſecond Shell will juſt lie in the Concavity of the firſt, and the third in the Concavity of the ſecond. The Solidity of the Metal of the firſt Shell is equal to its Concavity; and the Solidity of the Metal of the ſecond, to the Concavity, is as 7 to 5; and the Solidity of the third or leaſt Shell's Metal, to its Concavity, is as 9 to 4. Now, ſuppoſing the Diameter of the firſt, or greateſt Shell, to be 16 Inches, and allowing every ſolid Inch of Iron to weigh 4 Ounces; I demand the Diameter of the two leſſer Shells, and the Thickneſs and Solidity of Metal of every Shell, and alſo the Weight of every Shell.

The Cube of 16 is 4096: Then,

As 1 : .5236 :: 4096 : 2144.6656.

The Half thereof is 1072.3328, which is the Solidity of the Metal of the greater Shell, as alſo of the Concavity.

As .5236 : 1 :: 1072.3328 : 2048.

The Cube Root of 2048 is 12.699, which is the Diameter of the ſecond Shell.

The Sum of 7 and 5 is 12; then,

As 12 : 5 :: 1072.3318 : 446.805.

This 446.805 is the ſolid Content of the Concavity of the ſecond.

As .5236 : 1 :: 446.805 : 853.333.

The Cube Root of 853.333 is 9.485, the Diameter of the leaſt Shell.

The Sum of 9 and 4 is 13; then,

As 13 : 4 :: 446.805 : 137.47846.

This 137.47846 is the ſolid Content of the Concavity of the third.

As .5236 : 1 :: 137.47846 : 262.5639.

The Cube Root of 262.5639 is 6.4034, the Diameter of the leaſt Shell's Concavity.

From 16.. The Diameter of the greateſt,
Subtr. 12.699 The Diameter of the ſecond.

Rem. 3.301

Half is= 1.65 The Thickneſs of Metal of the greateſt.

From 12.699 The Diameter of the ſecond.
Subtr. 9.485 The Diameter of the leaſt.

Rem. 3.214

Half is= 1.607 The Thickneſs of Metal of the ſecond.

From 9.485 The Diameter of the leaſt.
Subtr. 6.403 The Diameter of the Concavity.

Rem. 3.082

Half is= 1.541 The Thickneſs of Metal of the leaſt.

The Metal of the greateſt is 1072.33 ſolid inches; which divide by 4, (becauſe every ſolid Inch is a Quarter of a Pound) the Quotient is 268.08 Pounds.

The Metal of the ſecond is 625.52 ſolid Inches; which divided by 4, the Quotient is 156.38 Pounds, the Weight of the ſecond.

The Metal of the leaſt Shell is 309.32 ſolid Inches; which divided by 4, the Quotient is 77.33 Pounds, the Weight of the leaſt.

The Diameter of the	ſecond Shell	12.699	Inches.
	leaſt Shell —	9.485	

The Thickneſs of the Metal of the	greateſt ——	1.65	Inches.
	ſecond ——	1.607	
	leaſt ——	1.541	

The Weight of the	greateſt —	268.08	Pounds.
	ſecond ——	156.38	
	leaſt ——	77.33	

A SHORT

BRITISH MUSEUM 15 APR 72

A SHORT

APPENDIX.

§I. *Of* GAGING, *according to both the* Iriſh *and* Engliſh *Gallon.*

I SHALL not here give the whole Art of Gaging (there being ſeveral Books of that Art already in Print, wrote by better Hands) but ſhall only lay down ſome ſhort practical Rules, whereby any Artificer, or others, may find the Quantity of Liquor in any Veſſel, upon Occaſion.

PROBLEM I.

To find the ſeveral Multipliers, Diviſors, and Gage-points belonging to the ſeveral Meaſures now uſed in England *and* Ireland.

282)1.0000(.003546 Multiplier for Ale Gallons.
231)1.0000(.004329 Multiplier for Wine Gallons.

268.8)1.000(.0037202 Multiplier for Corn Gallons.
2150.42)1.000(.00046502 Multiplier for Corn Buſhels.
217.6)1.0000(.0045955 Multiplier for *Iriſh* Gallons.

So, if the ſolid Inches in any Veſſel be multiplied by the ſaid Multipliers, the Product will be Gallons in the reſpective Meaſures: Or, dividing by the Diviſors 282, 231, 268.8, or 217.6, the Quotients will likewiſe be Gallons.

NOTE, That 282 ſolid Inches is a Gallon of Ale or Beer Meaſure; 231 ſolid Inches is a Gallon of Wine-Meaſure; 268.8 ſolid Inches is a Gallon, and 2150.42 ſolid Inches is a Buſhel of Corn-Meaſure. Alſo 217.6 ſolid Inches is a Gallon *Iriſh*, both of Ale, Wine, or Oyl.

For circular Areas, the following Multipliers and Diviſors are to be uſed.

282).785398(.002785 Multiplier for Ale Gallons.
231).785398(.003399 Multiplier for Wine Gal.
217.6).785398(.0036093 Multip. for *Iriſh* Gal.
.785398)282.(359.05 Diviſor for Ale Gal.
.785398)231.(294.12 Diviſor for Wine Gal.
.785398)2150.42(2738 Diviſor for Corn Buſhels.
.785398)217.6(277.05 Diviſor for *Iriſh* Gallons.

The Gage-point for Squares in	Ale-Meaſure,	is	16.79
	Wine-Meaſure,		15.19
	Malt Buſhel,		46 36
	Iriſh Gallons,		14.75

The Gage-point for circular Figures in	Ale Meaſure,	is	18 95
	Wine Meaſure,		17.15
	Malt Buſhel,		52.32
	Iriſh Gallon,		16.64

PROB-

PROBLEM II.

To find the Area in Ale, Wine, or Iriſh *Gallons, of any rectilineal plane Figure, whether Triangular, Quadrangular, or Multangular.*

TO reſolve this *Problem*, you muſt, by Chap. I. Part II. find the Area in Inches, and then bring it into Gallons, by dividing that Area in Inches by the proper Diviſor, *viz* by 282 for Ale, or by 231 for Wine, or 217.6 for *Iriſh*, or elſe by Multiplication, by 00.3546 for Ale, by .004329 for Wine, by .004595 for *Iriſh*, and the Quotient or Product will be the Area.

Example. Suppoſe a Back or Cooler in the Form of a Parallelogram, or Long Square, 250 Inches in Length, and 84.5 Inches in Breadth; what is the Area in Ale, Wine, or *Iriſh* Gallons.

Multiply 250 by 84.5, and the Product is 21125, the Area in Inches; which divided by 282, and the Quotient is 74.9 Gallons of Ale; or multiplied by .003546, the Product is 74.90925 Gallons, nearly the ſame; and if 21125 be divided by 231, or multiplied by .004329, it will give 91.45 Gallons of Wine; and if 21125 be divided by 217.6, or multiplied by .004595, it will give 97.06 *Iriſh* Gallons.

By Scale *and* Compaſſes.

Extend the Compaſſes from 282 to 250, that Extent will reach from 84.5 to 74.9: And

Extend from 231 to 250, that Extent will reach from 84.5 to 91.45.

Extend the Compaſſes from 217.6 to 250, that Extent will reach from 84.5 to 97.06.

NOTE, the Areas of all Superficies are always to be underſtood to be one Inch deep, otherwiſe it could not be ſaid that the Area of ſuch a Parallelogram, Circle, *&c.* is ſo many Gallons.

Having found the Area of a Back or Cooler, the next Thing will be to find out the true Dipping or Gaging-place in that Back, that ſo the true Quantity of Worts may be computed at any Depth; which may be thus done,

1. When the Bottom of the Back is covered all over (of any Depth) with Worts or other Liquor, then dip it in eight or ten ſeveral Places, (more or leſs, according to the Largeneſs of the Back) as remote and equally diſtant from each other as you can well do, noting down the wet Inches and Decimal Parts of every Dip.

2. Divide the Sum of all thoſe Dips by the Number of Places you dipped in, and the Quotient will be the mean Wet of all thoſe Dips.

3. *Laſtly*, find out ſuch a Place by the Side of the Back (if you can) that juſt wets the ſame with that mean Dip, and make a Notch or Mark there for the true and conſtant Dipping place of that Back.

Then, if any Quantity of Worts (which covers the whole Back) be dipped or gaged at that Place, and the wet Inches ſo taken be multiplied into the Area of the Back in Gallons, the Product will ſhew how many Gallons of Wort are in that Back at that Time, provided the Sides of the Back do ſtand at Right-angles with the Bottom.

PROBLEM III.

The Diameter of a Circle being given in Inches, to find the Area thereof in Ale, Wine, or Iriſh *Gallons.*

IF the Square of the Diameter be multiplied by .002785 for Ale, or by .003399 for Wine, or by .0036093 for *Iriſh* Gallons; or if it be divided by 359.05 for Ale, or by 294.12 for Wine, or by 277.05 for *Iriſh* Gallons, the

Products or Quotients will be the reſpective Ale, Wine, or *Iriſh* Gallons.

Example. Suppoſe the Diameter of a Circle be 32.6 Inches, what will be the Area in Ale, Wine or *Iriſh* Gallons?

The Square of 32.6 is 1062.76.

Then 359.05) 1062.76(2.9599 Area in Ale Gal.
And 294.12) 1062.76(3.6133 Area in Wine Gal.
And 277.05) 1062.76(3.8565 Area in *Iriſh* Gal.
Or 1062.76×.002785=2.9599 Ale Gal.
And 1062.76×.003399=3.6133 Wine Gal.
And 1062.76×.003609=3.8565 *Iriſh* Gal.

By Scale *and* Compaſſes.

Extend the Compaſſes from 18.95 (the Gage-point for Ale) to 32.6 (the Diameter) that Extent will reach from 1 to a fourth Number, and from that fourth to 2.9599 Gallons. Or, extend the Compaſſes from 1 to 32.6, that Extent, turned twice over from .002785, will at laſt fall upon 2.9599.

For Wine: Extend from 17.15 (the Gage-point for Wine) to 32.6 (the Diameter) that Extent, turned twice over from 1, will at laſt fall upon 3.6133 Gallons.

Or thus: Extend from 1 to 32.6, that Extent will reach from .003399 (being turned twice over) to 3.6133 Wine Gallons.

For *Iriſh* Gallons: Extend from 16.64 (the *Iriſh* Gage-point) to 32.6 (the Diameter) that Extent, turned twice over from 1, will at laſt fall upon 3.8565, the *Iriſh* Gallons ſought.

PROBLEM IV.

The Tranſverſe (or longeſt Diameter) and the Conjugate (or ſhorteſt Diameter of an Ellipſis (or Oval) being given, to find its Area in Ale, Wine, or Iriſh *Gallons*

IF the Rectangle, or Product of the two Diameters, that is, of the Length and Breadth of the Oval, be divided by 359.05, or multiplied by .002785 for Ale, or divided by 294.12, or multiplied by .003399 for Wine, or divided by 277.05, or multiplied by .0036093 the Quotients or Products will be the Ale. Wine or *Iriſh* Gallons required.

Example. Suppoſe the longeſt Diameter be 81.4 Inches, and the ſhorteſt Diameter be 54 6 Inches, what will be the Area of that Oval?

Multiply 81.4 by 54 6, and the Product is 4444.44; then

359.05)4444.44(12.38 Area in Ale Ga.
294.12)4444.44(15.11 Area in Wine Ga.
277.05)4444.44(16.71 Area in *Iriſh* Ga.
Or 4444.44×.002785=12.38 Ale Ga.
And 4444.44×.003399=15.11 Wine Ga.
And 4444.44×.003609=16.1 *Iriſh* Ga.

By Scale *and* Compaſſes.

Firſt, find a mean Proportional between 81.4 and 54.6, by dividing the Diſtance between them into two equal Parts, and the middle Point will be at 66.6, which is the mean Proportional, that is, the Diameter of a Circle equal to the Oval. Then extend the Compaſſes from 18.95 (the Gage-point for Ale) to 66.6, that Extent, turned twice over from 1, will at laſt fall upon 12.38 Ale Gallons: And extended from 17.15

(the Gage point for Wine) to 66.6, that Extent, turned twice over from 1, will reach at laſt to 15.11 Wine Gallons.

Laſtly, Extend the Compaſſes from 16.64 (the *Iriſh* Gage-point) to 66.6, that Extent, turned twice over from 1, will at laſt fall upon 16.1 *Iriſh* Gallons.

PROBLEM V.

To find the Content in Ale, Wine, or Iriſh *Gallons, of any Priſm, what Form ſoever its Baſe is of.*

FIRST find its ſolid Content in Inches, (by Sect. 1, 2. 3, of Chap. II. Part II.) then divide that Content in Inches by 282 for Ale, or by 231 for Wine, or by 217.6 for *Iriſh*; the reſpective Quotients will be the Content in Wine, Ale, or *Iriſh* Gallons.

Otherwiſe, you may find the Content of a Priſm, by finding the Area of its Baſe in Gallons, (by *Problem* II. of this Appendix) and multiply that Area by the Tun's Height or Depth within, the Product will be its Content in Gallons.

Example. Suppoſe a Tun, whoſe Baſe is a Parallelogram right-angled, its Length being 49.3 Inches, its Breadth 36.5 Inches, and the Depth of the Tun is 42.6 Inches; the Content in Ale, Wine, and *Iriſh* Gallons is required.

The Length, Breadth and Depth bejng multiplied continually, the Product is 76656 57; which divided by 282, the Quotient is 271.83 Ale Gallons; and divided by 231, the Quotient is 331.84 Wine Gallons; and divided by 217.6, the Quotient is 252.23 *Iriſh* Gallons; and by dividing by 2150.4, ſuch a Ciſtern will be found to hold 35.65 Buſhels of Corn.

By Scale *and* Compaſſes.

Extend the Compaſſes from 282 to 36.5, (the Breadth of the Baſe) that Extent will reach from 49.3 (its Length,) to 6.38 Ale Gallons, the Area of the Baſe. Then extend from 1 to 42.6, (the Depth) that Extent will reach from 6.38 (the Area of the Baſe) to 271.8 Gallons, the Content.

* * *

PROBLEM VI.

To find the Content of a Tun, whoſe Baſes are alike and parallel, but unequal, being the Fruſtum of a Pyramid.

FIND the Area of each Baſe, and a mean Proportional between them, and multiply the Sum of thoſe three by one third Part of the Depth or Height, and the Product is the Content.

Example. Suppoſe a Tun whoſe Baſes are Parallelograms, the Length of the greater is 100 Inches, and its Breadth 70 Inches; the Length of the leſſer Baſe 80, and its Breadth 56; and the Depth of the Tun 42 Inches; the Content in Ale and Wine Gallons is required.

Multiply 100 by 70, the Product is 7000, the Area of the greater Baſe; and 80 multiplied by 56, the Product is 4480, the Area of the leſſer Baſe; then multiply the two Areas into each other, and the Product is 31360000, whoſe ſquare Root is 5600, a geometrical mean Proportional.

The

The greater Area	7000	add
The leſſer Area	4480	
The mean Proportional	5600	
	17080	
A Third of the Depth	14	
	68320	
	17080	

282)239120(847.94 Ale Ga.
231)239120(1035.15 Wine Ga.
217.6)239120(1098.9 *Iriſh* Ga.

PROBLEM VII.

To find the Content of a Tun, whoſe Baſes are parallel and circular, being the Fruſtum of a Cone.

YOU may find the Content, as in the laſt *Problem*, by multiplying the Sum of the Areas of the two Baſes, and a mean Proportional, by one third Part of the Depth.

But it will be a ſhorter Way to find the Area of a mean Circle in Gallons, and multiply that by the Depth, thus: To the Rectangle of the greater and leſſer Diameters, add one third Part of the Square of the Difference of the Diameters, that Sum is the Square of a mean Diameter; which divided by 359.05 for Ale, or by 294.12 for Wine, or by 277.05 for *Iriſh* Gallons, gives the Area of a mean Circle in Ale, Wine, or *Iriſh* Gallons; which multiplied by the Depth, gives the Content.

Example. Suppoſe the greater Diameter 80 Inches, and the leſſer Diameter 71 Inches, and the Depth 34 Inches; the Content in Ale, Wine, and *Iriſh* Gallons is required.

Multiply 80 by 71, and the Product is 5680; to which add 27, (a third Part of the Square of the Difference of the Diameters) and the Sum is 5708, which is the Square of a mean Diameter; which divide by 359.05, and the Quotient will be 15.895 Ale Gallons the Area; which multiply by 34, (the Depth) and the Product will be 540.43 Gallons, the Content.

By Scale *and* Compaſſes.

Add the two Diameters together, and take half the Sum, which is 75.5, which take for a mean Diameter; (tho' it is not exact, yet it will be near enough the Truth, if the Difference between the Diameters be not great) extend the Compaſſes from 18.95, the Gage-point for Ale, to 75.5, the mean Diameter, that Extent will reach from 34, the Depth, to a fourth Number, and from that to 540.4 Gallons, the Content.

And if you extend the Compaſſes from 17.15, the Gage point for Wine, to 75.5, that Extent will reach from 34 (twice turned over) to 659.7 Gallons of Wine.

Laſtly, If you extend the Compaſſes from 16.64, the *Iriſh* Gage-point, to 75.5, that Extent will reach from 34 (twice turned over) to 700.3 *Iriſh* Gallons.

The Method uſed by the Gagers for all ſuch Tuns, is to take the Diameter in the Middle of every 10 Inches, that is, at five Inches from the Bottom, and at 15, and at 25, &c.

Then they find the Area to every one of theſe Diameters, and enter them in their Books: Then, when they ſurvey, they take the wet Inches and Parts that the Liquor in the Tun is in Depth, and every 10 Inches they take the reſpective Areas, and remove the ſeparating Point one Place towards the Right Hand, and for what odd Inches of the Depth above the even Tens, they multiply the next Area by them, and ſo add all the ſeveral Products together, and the Total will be the Gallons of Liquor in the Tun.

Example. Suppoſe the Diameter at 5 Inches from the Bottom be 64 Inches, and at 15 Inches from the Bottom 67 Inches, and at 25 Inches 70 Inches, and at 35 Inches from the Bottom, the Diameter is 73 Inches. Now the Area anſwering to 64 Inches, is 11.4078 Ale Gallons, and to 67 Inches is 12.5023 Gallons; and the Area to 70 Inches is 13.647 Gallons, and to 73 is 14.8418 Gallons: Then, ſuppoſing the Depth of the Liquor in the ſaid Tun be found to be 33.6 Inches; now, to caſt up this Gage, firſt, in the Area anſwering to 64 Inches, being multiplied by 10, that is, by removing the ſeparating Point a Place towards the Right Hand, it will be 114.078 Gallons, and the next will be 125.023, and the next 136.47 Gallons: Now theſe three will be the Content to 30 Inches deep. Then, to find the Content of the 3.6 Inches, multiply the next Area 14.8418 by 36, and the Product is 53.4305; add all theſe together, and the Sum is the whole Quantity of Liquor in the Tun.

The Content at 10 Inches deep	114.078
The Content at the next 10 Inches	125.023
The Content of the next 10 Inches	136.470
The Content of the 3.6 Inches	53.430
The whole Quantity of Liquor in the Tun, Ale Ga.	429.001

PROBLEM VIII.

To find the Drip or Fall of a Tun.

SUPPOSE the Tun laſt mentioned was ſo placed, that when the Bottom is but juſt covered on one Side, the Liquor is four Inches deep on the Side oppoſite; how much muſt be allowed for the Fall of this Tun; that is, how much Liquor is there in the Tun?

The Diameter in the Middle of 4 Inches from the Bottom, is 61.6 Inches, and the Area anſwering thereunto is 10.568; which multiplied by 2, (that is half 4) the Product is 21.136 Ale Gallons; and ſo much Liquor will juſt cover the Bottom;

But ſuppoſe it was ſet ſo much on one Side, as to be 30 Inches deep on one Side, when the Liquor on the oppoſite

Side juſt cuts between the Bottom and Staves; how much Liquor will there be in the Tun?

Square the bottom Diameter, and multiply that Square by the top Diameter, and divide the laſt Product by the Sum of the Diameters, and to the Quotient add the Square of the bottom Diameter, and divide the Sum by 1077.15 for Ale, or by 882.36 for Wine, or by 831.15 for *Iriſh* Gallons, multiply the Quotient by the Depth, the Product is the Content.

The bottom Diameter of the forementioned Tun is 61 Inches, and the Diameter at 30 Inches from the Bottom is 71.5 Inches; the Square of 61 is 3721, which multiplied by 71.5, the Product is 266051.5; this divided by 132.5, the Sum of the Diameters, the Quotient is 2007.936; to which add 3721, (the Square of 61,) and the Sum will be 5728.936; this divided by 1077.15, and the Quotient is 5.3186; which multiplied by 30, the Depth, the Product is 159.558, the Gallons of Liquor in the Tun.

When the Fruſtum of a Cone, or Pyramid, is cut by a Diagonal Plane thro' the Extremities of the Diameters, as the Liquor in the Tun repreſents, ſuch Solid is called a Hoof. (*Vide Ward*'s Young Mathematician's Guide, Page 414.)

If it be the Hoof of a ſquare Fruſtum, inſtead of dividing by 1077.15, divide by 846 for Ale, or by 693 for Wine, or by 652.3 for *Iriſh* Gallons. All the reſt of the Work is the ſame.

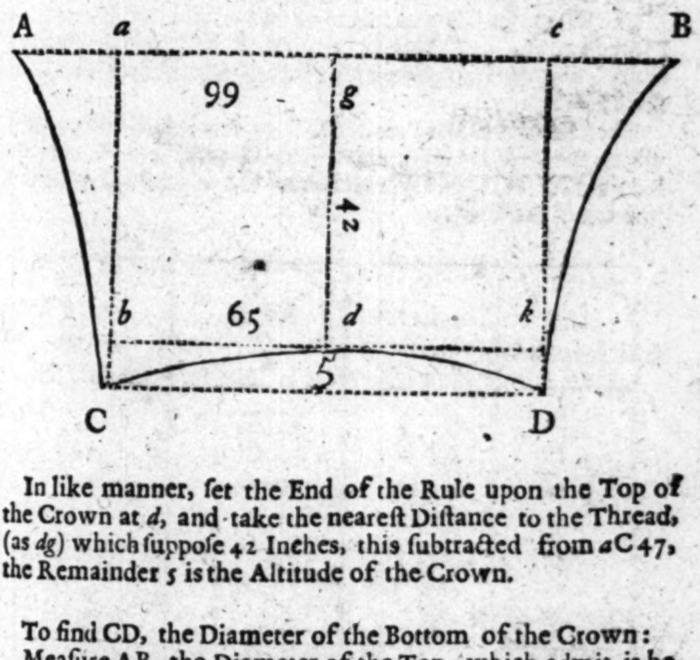
PROBLEM

PROBLEM IX.

To gage a COPPER.

LET ABCD be a ſmall Copper to be gaged.

Take a ſmall Cord, or Packthread, make one End faſt at A, and extend the other to the oppoſite Side of the Copper at B, where make it faſt, or cauſe ſome Perſon to hold it very ſtrait; then ſet one End of the Inſtrument in the Bottom of the Copper at C, and move it to and fro till you find the neareſt Diſtance to the Thread (as at *a*) this Diſtance, *a*C, is the Depth of the Copper, which ſuppoſe to be 47 Inches.



In like manner, ſet the End of the Rule upon the Top of the Crown at *d*, and take the neareſt Diſtance to the Thread, (as *dg*) which ſuppoſe 42 Inches, this ſubtracted from *a*C 47, the Remainder 5 is the Altitude of the Crown.

To find CD, the Diameter of the Bottom of the Crown: Meaſure AB, the Diameter of the Top, which admit it be 99 Inches, then hold a Thread ſo as a Plummer at the End thereof may hang juſt over C, by which Means you will find the Diſtance *a*A. Do the like on the other Side, ſo will you find alſo the Diſtance *c*B, which ſuppoſe 17.5 Inches each; add theſe two together, and ſubtract their Sum, *viz.* 35, from

59, and the Remainder is 64 Inches, the Diameter at the Bottom of the Crown: The Diameter which touches the Top of the Crown may be found by the Sliding-Rule to be 65 Inches.

Now, to find the Content of the Copper from the Crown upwards, that is, the Part AB*kh*, the Depth *gd* being 42 Inches, you may take a Diameter in the Middle of every 6 Inches of the Depth, which ſuppoſe to be as in the ſecond Column of the following Table, the Numbers in the third Column are the reſpective Areas in Ale Gallons, found by *Problem* III. the fourth Column ſhews the Content of every 6 Inches; all which being added together, the Sum will be the Content of that Part AB*kh*; that is, ſo much as it will hold after the Crown is covered.

Now, if the Crown be taken for the Fruſtum of a Sphere, the Content (by the latter Part of Sect. II. Page 179) will be found to be 28.75. Ale Gallons.

But may be more readily found, very near the Truth, thus: The Diameter CD was found to be 64, and the Area to this Diameter is 11.408; this multiplied by half the Crown's Altitude, *viz.* by 2.5, gives 28.52 Ale Gallons, the Content of the Crown.

The Content of the Part *hk*DC is 57.935 Ale Gallons, from which ſubtract the Content of the Crown, 28.52, and the Remainder is 39.415 Ale Gallons, and ſo much Liquor will juſt cover the Crown.

Parts of the Depth.	Diameters	Areas.	Content of every ſix Inches.
6	95.3	25.2945	151.767
6	90.1	22.6095	135.657
6	85.0	20.1223	120.734
6	80.	17.8246	106.947
6	75.2	15.7499	94.499
6	70.5	13.8426	83.056
6	66.	12.1319	72.791
		The Sum	765.451
		To juſt cover the Crown	29.415
		The whole Content in Ale Gallons	794.866

By

By Scale *and* Compaſſes.

You may find the Areas anſwering to every one of the Diameters, thus,

Extend the Compaſſes from the Gage-point to the Diameter, that Extent, being twice turned over from 1, will at laſt fall upon the Area of that Circle; or being turned twice over from 6, will give the Content of that 6 Inches of the Depth.

Example. Extend the Compaſſes from 18.95 (the Gage-point) to 95.3, that Extent, turned twice over from 6, will at laſt fall upon 151.76 Gallons, the Content of the firſt ſix Inches. And ſo of the reſt.

PROBLEM X.

To find the Content of any cloſe Cask.

IN order to perform this difficult Part of Gaging, the three following Dimenſions of the Cask muſt be truly taken,

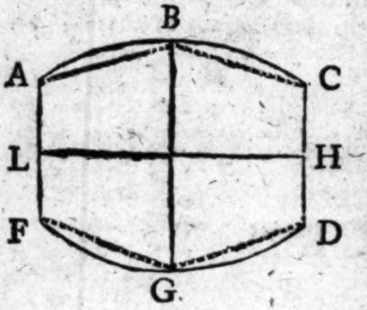
Viz. { The Bung-Diameter,
The Head Diameter,
The Length of the Cask, } within the Cask.

In taking theſe Dimenſions it muſt be carefully obſerved,

1. That the Bung-hole be in the Middle of the Cask; alſo, that the Bung-ſtaff, and the Staff oppoſite to the Bung-hole, are both regular, and even within.
2. That the Heads of the Cask are equal, and truly circular; if ſo, the Diſtance between the Inſide of the Chine to the Outſide of its oppoſite Staff, will be the Head-Diameter within the Cask, very near.
3. With a ſliding Pair of *Callipers* (made for that Uſe) take the ſhorteſt Diſtance, or Length, between the Outſides of the

two Heads; from that Length ſubtract $1\frac{1}{2}$ Inch, more or leſs, according to the Largeneſs of the Cask, for the Thickneſs of the Head; the Remainder will be the Length of the Cask within. But if the Cask be empty, you may take the Length by putting a ſtrait Rod in at the Tap-hole, and allow for the Thickneſs of the Head.

Now, by theſe Dimenſions, one would think the Content of the Cask was perfectly limited; but it will be eaſy to perceive, by the following Figure, that the Diameters and Length of one Cask may be equal to thoſe of another, and yet one of thoſe Casks may contain ſeveral Gallons more than the other.



As for Inſtance, the Figure ABCDF is ſuppoſed to repreſent a Cask: Then it is plain, that if the outward curved Lines, ABC and FGD, are the Bounds, or Staves of the Cask, it muſt needs hold more than if the inner pricked Lines were the Bounds or Staves; and yet the Bung-Diameter BG, and Head-Diameters CD and AF, and Length LH, are the ſame in both thoſe Casks.

Whence it appears, that no one general Rule can be given, whereby the Content of all Sorts of Casks can be gaged. And therefore Gagers do uſually ſuppoſe every Cask to be in ſome of theſe Forms;

1. The middle Fruſtum of a Spheroid.
2. The middle Fruſtum of a Parabolick Spindle.
3. The lower Fruſtums of two equal Parabolick Conoids.
4. The lower Fruſtums of two equal Cones.

1. If the Staves of the Cask be very much curved (as the outward Lines of the laſt Figure) then the Cask is ſuppoſed to be the middle Fruſtum of a Spheroid.

2. If the Staves between the Bung and the Head be ſomething leſs curved, then the Cask is taken to be the middle Fruſtum of a Parabolick Spindle.

3. If the Staves (between the Bung and Head) be very little curved, then the Cask is taken to be the lower Fruſtums of two equal Parabolick Conoids, abutting or joining together, upon one common Baſe.

4. If the Staves (between the Bung and Head) be ſtrait, as the pricked Lines in the laſt Figure, then the Cask is taken to be the lower Fruſtums of two equal Cones, abutting or joining together upon one common Baſe.

There are ſeveral Rules laid down in Books of Gaging, for finding the Content of each ſeveral Form, but I think the ſhorteſt and moſt practical Way is to find ſuch a mean Diameter, which will reduce the propos'd Cask to a Cylinder; thus.

Multiply the Difference of the Bung and Head Diameters by .7 for a Spheroid; by .65 for the ſecond Form, by .6 for the third Form, and by .55 for the fourth Form; and add the Product to the Head Diameter, and the Sum is a mean Diameter.

Example. Suppoſe the Bung-Diameter be 32 Inches, the Head-Diameter 24 Inches, and the Length 40 Inches; the Content in each Variety is required.

The Difference between the Bung and Head-Diameters is 8, which multiplied by .7, the Product is 5.6; which added to the Head-Diameter, the Sum is 29.6, the mean Diameter: The Area anſwering thereunto will be found (by Prob. III.) to be 2.44 Ale Gallons; which multiplied by the Length, the Product is 97.4 Gallons; and ſo much is the Content, if it be the firſt Form.

Again, If the Difference of the Diameters 8, be multiplied by .65, the Product will be 5.2; which added to the Head-Diameter, the Sum is 29.2 for the mean Diameter; and the Area anſwering thereunto is 2.3746 Gallons; which multiplied by 40, the Length, the Product is 94.98 Gallons, the Content, if it be of the ſecond Form.

Again, If the Difference 8, be multiplied by 6, the Product is 4.8; which added to the Head-Diameter, the Sum is 28.8, the mean Diameter; the Area thereunto is 2.31 Gallons; which multiplied by 40, gives the Content 92.4 Gallons, for the third Form.

Again, The Difference 8, multiplied by .55, the Product is 4.4; which added to the Head-Diameter, makes the mean Diameter 28.4; the Area thereof is 2.2463, which multiplied by 40, the Product is 89.85 Gallons, for the third Form.

By Scale *and* Compaſſes.

Extend the Compaſſes from the Gage-point 18.95, to the firſt mean Diameter 29.6, that Extent will reach from the Length 40, to a fourth Number, and then to the Content 97.4 Ale Gallons.

Again. Extend from 18.95 to 29.2, (the ſecond mean Diameter) that Extent, turned twice over from 40, will at laſt fall upon 94.98 Gallons.

Again, Extend from 18.95 to 28.8, (the third mean Diameter) that Extent, turned twice over from 40, will at laſt fall upon 92.4 Gallons.

Again, Extend from 18.95 to 28.4, (the fourth mean Diameter) that Extent, turned twice over from 40, will at laſt fall upon 89.85 Gallons.

Altho' I have all along made Uſe of the Line of Numbers upon the common two Foot or eighteen Inch Rules, for the Reaſon mentioned in the *Preface*; yet the Rules may eaſily be applied to the Sliding Rule, thus: To find the Area of a Circle in Gallons, ſet the Gage-point upon D, (that is, a ſingle Line of Numbers) to 1 upon C, (that is, a double Line) then againſt any Diameter upon D, is the Area upon C, thus:

To find the Content of the Cask laſt mentioned. The firſt Form:

Set the Gage-point 18.95 upon D, to the Length 40 upon C; then, againſt the mean Diameter 29.6 upon D, is 97.4 Gallons, the Content upon C.

And againſt 29.2 (the next mean Diameter) on D, is 94.98 Gallons on C.

And againſt 28.8 (the next mean Diameter) on D, is 92.4 Gallons on C.

And againſt 28.4 (the laſt mean Diameter) on D, is 89.85 Gallons on C.

All done without removing the Slider.

A TABLE

A TABLE *of the Segments of a Circle whose Area is Unity.*

V. S.	Segm.	V. S.	Segm	V. S.	Segm.	V. S.	Segm.
1	.0017	99	.9983	26	.2066	74	.7934
2	.0048	98	.9952	27	.2178	73	7822
3	.0087	97	.9913	28	.2292	72	.7708
4	0134	96	.9866	29	.2407	71	.7593
5	.0187	95	.9813	30	.2523	70	.7477
6	.0245	94	.9755	31	.2640	69	.7360
7	.0308	93	.9692	32	.2759	68	.7241
8	.0375	92	.9625	33	.2878	67	.7122
9	.0446	91	.9554	34	.2998	66	.7002
10	.0520	90	.9480	35	.3119	65	.6881
11	.0598	89	.9402	36	.3241	64	.6759
12	0680	88	9320	37	.3364	63	.6636
13	0764	87	.9236	38	.3487	62	.6513
14	0851	86	.9149	39	.3611	61	.6389
15	.0941	85	9059	40	.3735	60	.6265
16	.1033	84	.8967	41	.3860	59	.6140
17	1127	83	.8873	42	.3986	58	.6014
18	.1224	82	.8776	43	.4112	57	.5888
19	.1323	81	.8677	44	.4238	56	.5762
20	.1424	80	.8576	45	.4364	55	.5636
21	.1526	79	.8474	46	.4491	54	5509
22	.1631	78	.8369	47	.4618	53	5382
23	.1737	77	.8263	48	.4745	52	.5255
24	.1845	76	.8155	49	.4873	51	.5127
25	.1955	75	.8045	50	.5000	50	.5000

The Uſe of the Table of Segments.

§ 1. *Is to find the Ullage, or Quantity of Liquor remaining in a Caſk, whoſe Axis is parallel to the Horizon, the Surface of the Liquor cutting the Heads of the Caſk.*

The RULE *is.*

TO the wet or dry Inches of the Bung Diameter add a competent Number of Cyphers, then divide it by the whole Diameter, the Quotient found in the Table under the Title V. S. gives a Segment; which multiplied by the whole Content of the Cask, the Product ſhews the Quantity of Liquor in the Cask, if the Dividend was the wet Inches, or the Ullage, if it was the dry.

Let there be a Cask in form of a Cylinder, whoſe Bung-Diameter is 29 Inches, the dry Part 13, and the wet 16, and the Content 80 Gallons; how many Gallons are wanting to fill the Cask?

Divide the dry Inches 13, by 29 the Bung-Diameter, and the Quotient is .448; find the two firſt Figures .44 under V S. and the Segment againſt it is .4238, to which add a proportional Part for the 8, and the whole Segment will be .4333; which multiplied by the Content of the Cask, the Product will be 34 664 Gallons; and ſo much the Caſk wants of being full.

Note, If the Cask be in the form of a Cylinder, or near that Figure, the Table will give the Ullage exact enough; but if it be a ſpheroidal Cask, then uſe the following Method:

1. By the Bung and Head-Diameters, find ſuch a mean Diameter as you judge will reduce the propoſed Cask to a Cylinder, and then find its Content.

2. From the Bung-Diameter ſubtract the mean Diameter, and take half the Difference.

3. From the wet Inches ſubtract the ſaid half Difference, reſerve this Difference, then uſe this Proportion;

As the mean Diameter: is to 100,
(the Diameter of the tabular Circle) ::
ſo is the reſerv'd Difference:
to a verſed Sine in the Table.

Then, if the tabular Segment be multiplied into the Content (as before) the Product will be the Quantity of Liquor in the Cask.

Example. Let the Cask be the ſame as in Page 300 of the firſt Form, where the Bung-Diameter is 32 Inches, and the mean Diameter 29.6, and the Content 97.4 Gallons; and ſuppoſe the wet Inches 19, to find the Quantity of Liquor in the Cask.

From	32	From	19
Subtr.	29.6	Subtr.	1.2
Rem,	2.4	Rem.	17.8 reſerv'd.
Half	1.2		

As 29.6 : 100 :: 17.8 : 60, the V. S.

The Segment to 60 is .6265, which multiplied by 97.4, the Content, the Product is 61 Gallons, the Quantity of Liquor in the Cask.

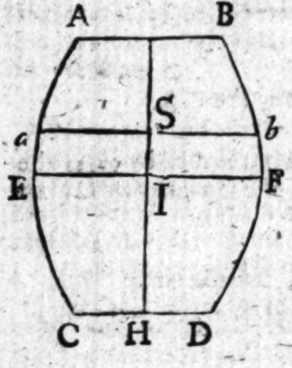
If the dry Inches had been given, by the ſame Method you might have found the Ullage, or what the Cask wanted of being full.

§ 2. *To find what Quantity of Liquor is in a Cask, when its Axis is perpendicular to the Horizon, viz. when it ſtands upright upon one of its Heads.*

To do this, you muſt know how to calculate the Area of any Circle, between the Bung and Head, whoſe Diſtance from the Bung, or Middle of the Cask is given; which may be done by this Proportion:

As the Square of half the Length of the Cask: is to the Difference between the Bung and Head-Areas : : ſo is the Square of any Circle's Diſtance from the Bung : to the Difference between the Bung-Area and the Area of that Circle, *viz.* the Area of the Liquor's Surface.

Then, from the Bung-Area ſubtract one third Part of the aforeſaid Difference, *viz.* between the Bung-Area and the Area of the Liquor's Surface; multiply the Remainder by the Liquor's Diſtance from the Bung, and the Product will ſhew what Quantity of Liquor is either above or under half the Content of the Cask.



Example. Let us again ſuppoſe the Cask in Page 300, whoſe Length is 40 Inches, Bung-Diameter 32, and Head-Diameter 24, and ſuppoſe the wet Inches SH 26 Inches.

The Square of half the Length is 400, the Diſtance of the Liquor's Surface from the Bung SI is 6, whoſe Square is 36; the Area of the Bung-Diameter 2.8519 Ale Gallons, and the Area of the Head-Diameter 1.6042; the Difference 1.2477. Then,

As 400 : 1.2477 : : 36 : .0751.

One Third is =.0250

From

From	2.8519	Bung Area,
Subtr.	.0250	A Third of the Difference.
Rem.	2.8269	
	6	Multip. Diſtance from the Bung.
	16.9614	Content above the Bung
And	48.7	Half the Content of the Cask.
	65.66	The Quantity of Liquor in the Cask.

PROBLEM XI.

Gaging of MALT.

TO find the Quantity of Malt in a Ciſtern, or upon a Floor.

Firſt, find the Area of the Baſe in Buſhels, by multiplying the Length by the Breadth, and dividing the Product by 2150.42, or only by 2150. and multiply that Area by the mean Depth: (how to take the mean Depth, ſee *Problem* II.) If the Baſe be circular, or oval, divide by 2738. (See *Problem* I.

Example There is a Ciſtern whoſe Length is 84 Inches, and Breadth 54 Inches, and the mean Depth is 43.6 Inches; what is the Content?

Multiply 84 by 54, and the Product is 4536; which divide by 2150, and the Quotient is 2.1097 Buſhels, the Area of the Bottom at 1 Inch deep; which multiplied by the Depth 43.6, and the Product is 91.98 Buſhels, the Content.

Example. Suppoſe a Quantity of Malt upon a Floor whoſe Length is 245 Inches, and the Breadth 184 Inches, and the mean Depth 5.6 Inches; how many Buſhels are there?

Multiply 245 by 184, and the Product is 45080, which divided by 2150, the Quotient is 20.967, the Area of the Baſe, which multiplied by the mean Depth, the Product is 117.4 Buſhels, the Content.

By the Sliding Rule.

There is an inverted Line of Numbers upon ſome Sliding-Rules, marked with the Letter M, which was contrived purpoſely for gaging of Malt; and there is a double Line of Numbers upon the Rule, and upon the Slider two double Lines of Numbers; all theſe are of equal Radius, and all work together at once: Thus, ſet the Length and Breadth againſt one another upon the inverted Line, and that which ſlides by it; then, on the other Edge of the Rule, againſt the Depth, you will find the Content in Buſhels: Thus, in the firſt *Example*, ſet 54 upon the Slider againſt 84, upon the inverted Line, and then, againſt 43.6, upon the other Part of the Rule, is 91.98 upon the Slider.

Again, in the ſecond *Example*, ſet 184 upon the Slider to 245 upon the inverted Line, and againſt 5.6 upon the other Part of the Rule is 117.4 upon the Slider.

* * *

§ II. *Of* Land-meaſuring.

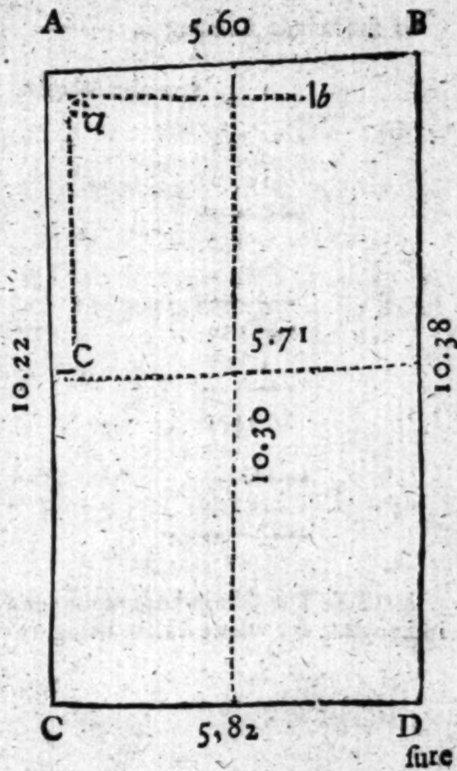
I SHALL not here give the whole Art of *Surveying*, but ſuch practical Rules only as may be uſeful to the Country Graſiers and Farmers, whereby they may find the true Content of any Piece of Land, and that by the Chain only; and, for want of that, with a Pole or Stick of half a Rod in Length.

PROBLEM I.

To find the Content of a Piece of Land, in the Form of a right-angled Parallelogram, or Long Square, or what is ſomething near that Form.

TO know whether any Angle in a Field be a Right-angle or not, you may take a Piece of Board about 4 or 5 Inches broad, and an Inch thick, either round or ſquare, and with a Saw cut two Kerfs, croſſing each other at Right-angles, and bore a Hole in the middle of the back Side, to put it upon the End of a Stick; this will repreſent the Inſtrument called a Croſs.

Suppoſe you would obſerve the Angle A, to know whether it be a Right-angle, (or near thereunto) prick up your Stick, with the Croſs upon it, a little Diſtance from the Fence, as at *a*, and having ſet up two Marks, as at *b* and *c*, of equal Diſtance from the Fence, turn one of the Slits directly towards *b*; and then, if the other be directly pointing to *c*, it is a Right-angle.



To meaſure ſuch a Piece of Ground as this Figure above: If you meaſure round, and add the oppoſite Sides together, and take half the Sum, (if they be not equal) or elſe meaſure down about the Middle of the Length and Middle of the Breadth, thus, the Side AB being meaſured, it will be 5.60, that is, 5 Chains and 60 Links; and the oppoſite Side CD is 5 Chains 82 Links; the half Sum thereof is 5.71: And the Side BD is 10.38, and the Side AC 10.22; and the half Sum thereof is 10.30: (it will be the ſame Thing if you meaſure about the Middle of the Length and Middle of the Breadth) then multiply this mean Length and mean Breadth together, *viz.* 10.30, by 5.71, and the Product is 58.8130; which divide by 10, (becauſe 10 ſquare Chains is an Acre) by removing the ſeparating Point one Place towards the Left Hand, and it will be 5.88130, that is, 5 Acres and .88130 Parts; which multiply by 4, and prick off 5 Places, and it will be 3.52520; which 3 towards the Left Hand are 3 Roods; then multiply the decimal Parts by 40, and prick off 5 Places, and it will be 21.00800; which 21 towards the Left Hand are 21 Perches.

	A.	R.	P.
So the whole Content is ———	5	3	21

See the Work.

	A.	R.	P.
5.71 10.30	5 :	3 :	21
17130 571			
5.88130 4			
3.52520 40			
21.00800			

NOTE, The Chain here made uſe of is 4 Poles, or Rods, in Length; the whole Chain being 100 Links.

But becauſe every Man that may have Occaſion to meaſure a Piece of Land can't procure a Chain, I will therefore ſhew how you may meaſure a Piece of Land only with a Stick of half a Rod in Length, that is, 8 Feet and 3 Inches; but in *Ireland* 'tis 10 Foot 6 Inches; which Stick divide into five equal Parts, ſo will the whole Rod be divided into ten Parts, and will be thereby adapted to decimal Arithmetick.

But becauſe each of thoſe Parts of the Stick are ſomething large, (each Part being 19 Inches and 8 Tenths in *Great-Britain*, but 25.2 Inches in *Ireland*) it will be neceſſary to take your Dimenſions to one half of thoſe Parts, and then, for that half Part. ſet 5 in the Place of Seconds, thus; ſuppoſe 3 Parts and a half, ſet it down thus, .35.

PROBLEM II.

LET us ſuppoſe a Field in the Form of a Long Square, whoſe Length is 45 Rods, 5 Parts and a half, and the Breadth 31 Rods, 4 Parts and a half; what is the Content?

Multiply the Length and Breadth together, and divide the Product by 150, (becauſe 160 ſquare Rods are in an Acre) and the Quotient is Acres.

```
     45.55
     31.45
   -------
     22775
    18220
    4555
  13665
  --------
  1432.5475
```

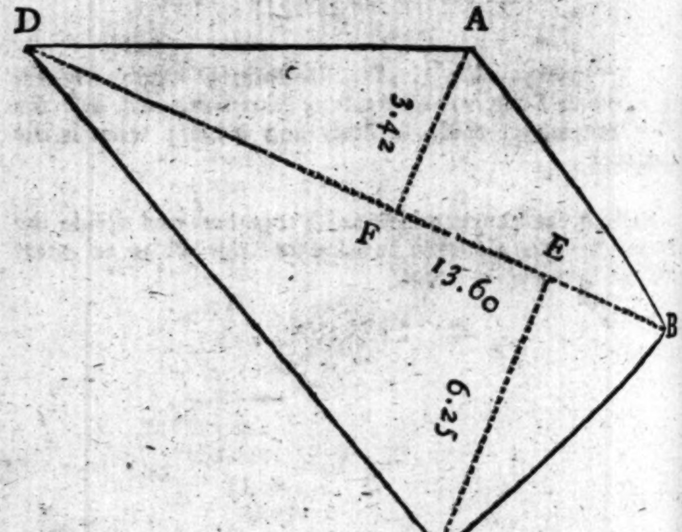
160)

```
16|0)143|2(8
     128
    ----
   4|0)15|2(3
         12
        ----
         32
```

A. R. P.
Facit 8 3 32

PROBLEM III.

SUPPOSE a Piece of Ground in the Form of a Trapezium, the Diagonal BD 13 Chains 60-Links, the Perpendicular CE 6 Chains 25 Links, and the Perpendicular AF 3 Chains 42 Links; what is the Content?



Multiply the Diagonal by half the Sum of the Perpendiculars. *See Sect.* VI. *of Chap.* I. *Part* II.

```
CE=6.25          13.60=BD
AF=3.42           4.83
   -----         ------
Sum 9.67          4080                A.  R.  P.
   -------       10880         Facit 6    2   11
Half 4.83        5440
                 -------
                 6.56880
                       4
                 -------
                 2.27520
                      40
                 -------
                 11.00800
                 -------
```

By Rods, thus.

```
CE=25    Rods.               19.34
AF=13.68                      54.4=BD
   -----                    -------
Sum 38.68                     7736
   -----                     7736
Half 19.34                  9670
                           -------
                       16|0)105|2.096(6
                             96
                            -------
                          4|0)9|2(2
                              8
                             ---
                             12

       A.  R.  P.
Facit 6    2   12
```

To take the Dimenſions of the Field.

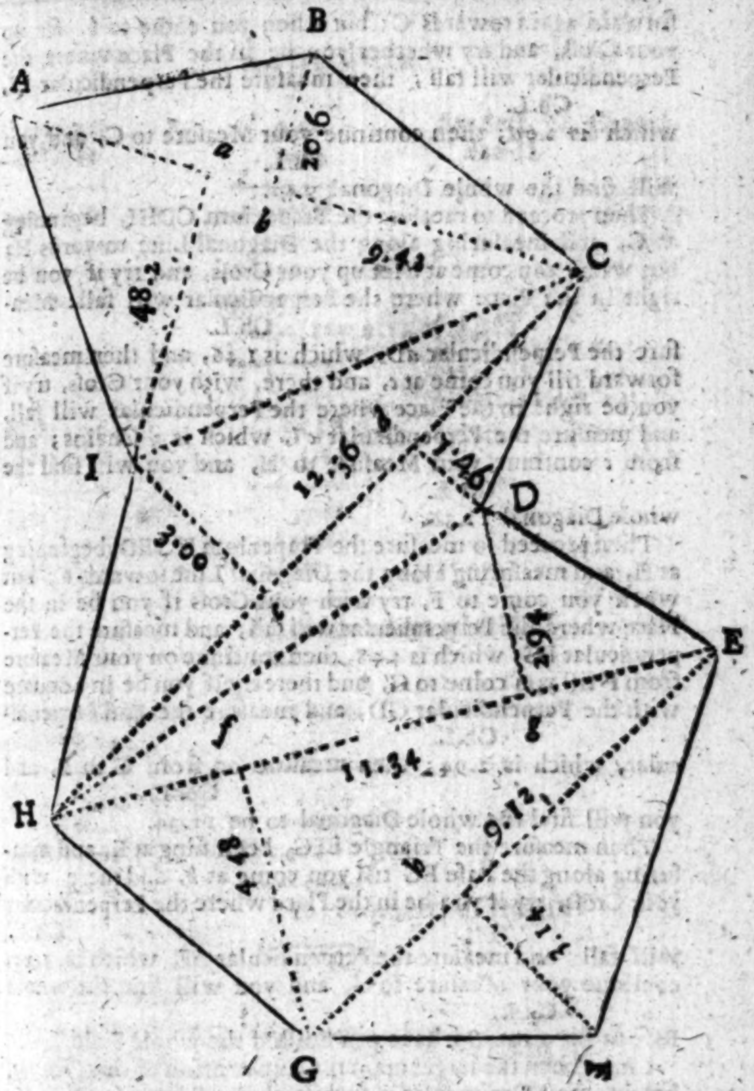
Begin at the Angle B, and meaſure in a direct Line toward D; but when you come at E ſet up your Croſs, and direct one of the Slits to D, and then look through the other Slit, and if it exactly hits the Angle C, then you are juſt in the Place where the Perpendicular will fall; but if it does not exactly hit the Point, move backwards or forwards till it does ſo; then meaſure the Perpendicular, and ſet down the Chains and Links, or the Rods and Parts; then continue your Meaſure towards D; but when you come to F, ſet up your Croſs, and try (as above directed) whether you be in the Place where the Perpendicular will fall: Then meaſure the Perpendicular AF, and ſet down the Chains and Links, or Rods and Parts; then continue your Meaſure to D, and ſet down the Meaſure of the whole Diagonal. This Way of meaſuring is very exact and true; but the common Way uſed by the Graſiers and Farmers, is to meaſure round the Field, and to take half the Sum of the oppoſite Sides for a mean Side; but the laſt mentioned Piece of Ground being meaſured ſo, will come to
A. R. P. R. P.
7 0 22, which is 2 10 more than the Truth.

PROBLEM IV.

How to meaſure an irregular Field.

THE Way to meaſure irregular Land, is to divide it into Trapeziums and Triangles; thus,

Firſt, view over the Field, and ſet up Marks at every Angle, and by thoſe Marks you may ſee where to have a Trapezium, as ABCI in the following Figure.



Then begin and meaſure in a direct Line from A towards C; but when you come to *a* ſet up your Croſs, and try whether you be in a Square to I, (as is before directed) and then
Ch.L.
meaſure the Perpendicular *a*I, which is 4.82; then meaſure forward

forward again towards C; but when you come to *b*, ſet up your Croſs, and try whether you be in the Place where the Perpendicular will fall; then meaſure the Perpendicular *b*B,
Ch.L.
which is 2.06; then continue your Meaſure to C, and you
Ch.L.
will find the whole Diagonal 9.42.

Then proceed to meaſure the Trapezium CDHI, beginning at C, and meaſuring along the Diagonal Line towards H; but when you come at *d* ſet up your Croſs, and try if you be right in the Place where the Perpendicular will fall: Mea-
Ch.L.
ſure the Perpendicular *d*D, which is 1.46, and then meaſure forward till you come at *c*, and there, with your Croſs, try if you be right in the Place where the Perpendicular will fall, and meaſure the Perpendicular *c* I, which is 3 Chains; and from *c* continue your Meaſure to H, and you will find the
Ch.L.
whole Diagonal 11.36.

Then proceed to meaſure the Trapezium HGED, beginning at H, and meaſuring along the Diagonal Line towards E; but when you come to F, try with your Croſs if you be in the Place where the Perpendicular will fall; and meaſure the Perpendicular FG, which is 4.48, then continue on your Meaſure from F till you come to G, and there try if you be in a Square with the Perpendicular GD; and meaſure the ſaid Perpendi-
Ch.L.
cular, which is 2.94; then meaſure on from G to E, and
Ch. L.
you will find the whole Diagonal to be 11.34.

Then meaſure the Triangle EFG, beginning at E, and meaſuring along the Baſe EG till you come at *h*, and there, with your Croſs, try if you be in the Place where the Perpendicular
Ch.L.
will fall; and meaſure the Perpendicular *h*F, which is 3.14, continue your Meaſure to G, and you will find the whole
Ch.L.
Baſe to be 9.12: So have you finiſh'd the whole Field.

I have been the larger upon the Explanation of this *Problem*, becauſe moſt Grounds lie in ſuch irregular Forms.

Caſt up the three Trapeziums ſeverally, and alſo the Triangle, and add all the ſeveral Areas together into one Sum, which will be the Area of the whole irregular Plot.

See the Work.

bB=2.06	9.42	See Sect. VI. Chap. I.
aI=4.82	3.44	Part II.
Sum 6.88	3768	
	3768	
Half 3.44	2826	
	3.24048=Area of ABCI.	

dD=1.46	12.36
cI=3.00	2.23
Sum 4.46	3708
	2472
Half 2.23	2472
	2.75628=Area of CIHD.

FG=4.48	11.34
GD=2.94	3.71
Sum 7.42	1134
	7938
Half 3.71	3402
	4.20714=Area of HGED.

Baſe=9.12	
Half=4.56	See Sect. V. Chap. I.
Perpend. 3.14	Part II.
1824	
456	
1368	

1.43184=Area of the Triangle EFG.

1.43184

forward again towards C; but when you come to *b*, ſet up your Croſs, and try whether you be in the Place where the Perpendicular will fall; then meaſure the Perpendicular *b*B,
Ch.L.
which is 2.06; then continue your Meaſure to C, and you
Ch.L.
will find the whole Diagonal 9.42.

Then proceed to meaſure the Trapezium CDHI, beginning at C, and meaſuring along the Diagonal Line towards H; but when you come at *d* ſet up your Croſs, and try if you be right in the Place where the Perpendicular will fall: Mea-
Ch.L.
ſure the Perpendicular *d*D, which is 1.46, and then meaſure forward till you come at *c*, and there, with your Croſs, try if you be right in the Place where the Perpendicular will fall, and meaſure the Perpendicular *c* I, which is 3 Chains; and from *c* continue your Meaſure to H, and you will find the
Ch. L.
whole Diagonal 12.36.

Then proceed to meaſure the Trapezium HGED, beginning at H, and meaſuring along the Diagonal Line towards E; but when you come to F, try with your Croſs if you be in the Place where the Perpendicular will fall; and meaſure the Perpendicular FG, which is 4.48, then continue on your Meaſure from F till you come to G, and there try if you be in a Square with the Perpendicular GD; and meaſure the ſaid Perpendi-
Ch.L.
cular, which is 2.94; then meaſure on from G to E, and
Ch. L.
you will find the whole Diagonal to be 11.34.

Then meaſure the Triangle EFG, beginning at E, and meaſuring along the Baſe EG till you come at *h*, and there, with your Croſs, try if you be in the Place where the Perpendicular
Ch.L.
will fall; and meaſure the Perpendicular *h*F, which is 3.14, continue your Meaſure to G, and you will find the whole
Ch.L.
Baſe to be 9.12: So have you finiſh'd the whole Field.

I have been the larger upon the Explanation of this *Problem*, becauſe moſt Grounds lie in ſuch irregular Forms.

Caſt up the three Trapeziums ſeverally, and alſo the Triangle, and add all the ſeveral Areas together into one Sum, which will be the Area of the whole irregular Plot.

See the WORK.

*b*B=2.06 9.42 See Sect, VI. Chap. I.
*a*I=4.82 3.44 Part II.

Sum 6.88 3768
3768
Half 3.44 2826

3.24048=Area of ABCI.

*c*D=1.46 12.36
*c*I=3.00 2.23

Sum 4.46 3708
2472
Half 2.23 2472

2.75628=Area of CIHD.

FG=4.48 11.34
GD=2.94 3.71

Sum 7.42 1134
7938
Half 3.71 3402

4.20714=Area of HGED.

Baſe=9.12

Half=4.56 See Sect. V. Chap. I.
Perpend. 3.14 Part II.

1824
456
1368

1.43184=Area of the Triangle EFG.

1.43184

1.43184 = Area of the Triangle EFG.
3.24048 = Area of ABCI.
2.75628 = Area of CIHD.
4.20714 = Area of HGED.

Sum 11.63574 = Area of the Whole.
4

2.54296
40

21.71840

	A.	R.	P.
Facit	11	2	21

F I N I S.

CORRIGENDA.

In Page 60, last two Lines.

For

$$\frac{8\quad 1\ 10}{12677\quad 8\ \ 8}\ \Big|\ \tfrac{1}{6}\text{ of }48\ 11$$

Read

$$\frac{8\quad 0\ \ 0}{12677\quad 6\ 10}\ \Big|\ \tfrac{1}{6}\text{ of }48\text{ Feet.}$$

Page 115. For 10.25 half the Baſe MA.
Read 10.25 = MA, half the Baſe.

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